Second-digit Tests for Voters' Election Strategies and Election Fraud*

Walter R. Mebane, Jr.†

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†Professor, Department of Political Science and Department of Statistics, University of Michigan, Haven Hall, Ann Arbor, MI 48109-1045 (E-mail: wmebane@umich.edu).

Abstract

The second significant digits of precinct-level (or polling station-level) vote counts follow regular patterns when voters are acting strategically and when there are partisan imbalances of voters in districts (when there is a gerrymander). The digits often follow distinctive patterns when vote counts are affected by coercion. The patterns are illustrated by a simulation exercise that generates individual preferences that, when aggregated into precincts, have counts whose second but not first significant digits approximately satisfy Benford's Law (2BL). Deriving sincere, strategic, gerrymandered and coerced votes from these preferences under a plurality voting rule shows that the second digits of the precinct counts are sensitive to differences in how the counts are derived. The patterns in the simulations are similar to those in real data from German Bundestag elections and from American presidential, House and state legislative elections. The strategic and gerrymandering effects are well known in both the German and American elections—the effects in the American national elections are associated with strategic party balancing between the president and the legislature—so overall I offer an inductive argument that the vote counts' digits are diagnosing the same kinds of behavior in both the simulated and real election data. Covariates such as the margin between candidates are important, but digits can detect strategic behavior even without having any information about preferences or beliefs. The 2BL regularity is not enough, but against the more nuanced background of regular digit patterns, election fraud in the form of coercion can stand out.

Introduction

Voting is complicated, and diagnosing whether something is wrong with the vote count in an election should take the complications into account. Among the primary complications any diagnostic scheme needs to acknowledge are strategic voting and gerrymandering. Strategic voting refers to the fact that when voters take the preferences, beliefs and likely behavior of other voters into account, many may cast votes that differ from what they would do if they acted based solely on their own preferences. Gerrymandering refers to the fact that often in drawing legislative districts imbalances are created so that one party has a systematic advantage. The term "gerrymandering" usually suggests intentional manipulation (Cox and Katz 2002), but imbalances may be created inadvertently, perhaps reflecting transient opinions rather than longstanding partisan divisions.

Under assumptions that many voters behave rationally, theory has been developed to describe the consequences of strategic behavior in many circumstances, including for example "wasted vote logic" and its connection to Duverger's Law (Cox 1994, 1996; Chhibber and Kollman 2004), strategic coordination that links together votes for president and for legislative offices (Alesina and Rosenthal 1995, 1996; Mebane 2000; Mebane and Sekhon 2002) and "threshold insurance" and other coalition-aware voting strategies (Bawn 1999; Shikano, Herrmann and Thurner 2009; Schofield and Sened 2006). The particular kinds of strategies voters may use are many and varied, depending on many contingencies specific to each situation. But in all cases the strategies have in common that voters consider their preferences and their beliefs about what others will do, given the election rules and other circumstances in effect, and then each voter takes the action that seems best to him or her.

Such strategic behavior differs fundamentally from what I'll call *coercion*, a situation in which all votes are cast in a way that reflects some single person's intention.[1] Voters acting

[1] Some mention legal and other social considerations to define election fraud (Lehoucq 2003). I focus on trying to detect coercion.

in line with threats or bribes are being coerced, and simple ballot box stuffing also counts as coercion: the ballots all reflect the will of whomever suborned the phony votes. Strategic votes remain heterogeneous in ways that coerced votes do not. Only some voters who consider modifying their behavior in light of strategic considerations do so—only those who have particular preferences and beliefs do things differently than they would in the absence of strategic considerations—while a much higher proportion of voters who are coerced act in a different way because of the coercion. Many preference and belief configurations map onto a small number of actions when voting is strategic, while the same range of preferences and beliefs all map onto one action under coercion. Perhaps the differences between these mappings cause distinctive patterns in the vote counts that can be detected using only the vote counts, even when information about preferences and beliefs is lacking.

Here I'm concerned with tests that can be used to diagnose election irregularities in the absence of information about preferences or beliefs. Whether such diagnosis is possible is of course a question, but some claim that some preference-free diagnostic methods can detect problems (Pericchi and Torres 2004; Mebane 2006, 2008; Mebane and Kalinin 2009; Mebane 2010*b*). The referent tests don't use any information about preferences, but instead look at patterns in the second significant digits of precinct vote counts. In early work using such information (e.g. Pericchi and Torres 2004; Mebane 2006), the idea was that if the distribution of those digits differs significantly from the one implied by Benford's Law—so-called second-digit Benford's Law (2BL) tests—then probably there is something wrong with the election and investigation using much richer kinds of information is warranted. Some of these methods use modified versions of Benford's Law (Pericchi and Torres 2011; Shikano and Mack 2009). Some have expressed skepticism regarding the relevance of Benford's Law in this context (López 2009; Deckert, Myagkov and Ordeshook 2011; Mebane 2011). The important issue is whether this kind of test can distinguish irregularities from strategic voting and from gerrymandering. To put it a little more sharply, can the tests distinguish election fraud from normal politics?

Strategic behavior can produce results that are surprising if one knows about voters' preferences but not about their beliefs or strategies. Some candidates may receive many more votes than preferences alone would indicate, while others surprisingly receive very small or even negligible shares of the vote. Allegations that there are irregularities in vote counts may seem plausible in such circumstances if the possibility that there was strategic voting is ignored.

Likewise different groupings of voters into constituencies—different gerrymanders—can produce different election outcomes even if individual voters' preferences don't change under different ways of drawing district lines, if only because of rolloff. A lopsided partisan gerrymander may prompt some voters not to participate in the election, just because the election outcome is not in doubt or because of varying mobilization actions taken by elites (Cox and Munger 1989; Berch 1989). As the number of votes parties receive change we should expect the pattern of digits in the vote counts to change as well. The districting decisions that produce such changes are also not fraud.

Mebane (2008) concluded that "as measured by the 2BL test, signs of election fraud in recent American presidential votes seem to be rare." As I will demonstrate below, while this impression may be correct substantively—there is not a lot of fraud—it lacks nuance as a technical matter. A different statistic than was used in Mebane (2008) shows extensive and significant departures from the 2BL pattern in American elections during both the 1980s and the 2000s. The departures affect not only votes recorded for president but for other federal offices such as the U.S. House of Representatives. Election returns for state-level offices, such as votes for state legislative seats, similarly fail to follow the basic 2BL distribution. The discrepancies from 2BL are similar across all these offices. The departures reflect the effect that strategic voting and gerrymandering have on 2BL tests.

After reviewing some basic definitions for 2BL test statistics, I start with two examples taken from American election data and two from German election data that help motivate the current analysis. Then I present a set of Monte Carlo simulation studies that illustrate the different effects strategic voting, gerrymandering and coercion have on the distribution of second digits in vote counts. It will appear that in conjunction with covariates that may be derived from the vote counts themselves, and not using any additional information, tests on the second digits of precinct-level vote counts can detect strategic voting and gerrymandering. Then I examine data from elections in Germany and the United States, to show that many of the patterns that appear in the simulations also appear in real election data. Strategic voting behaviors and gerrymanders affect the real election data in ways that for the most part have previously been written about, so my inductive method is to show that in elections where it is already well understood that gerrymanders and voters' strategies affect the results, tests based on the second digit of precinct-level (or polling station-level) vote counts also exhibit patterns that are distinctive and match what the simulation results show.

2BL Test Statistics

Benford's Law describes a distribution of digits in numbers that arises under a wide variety of conditions. Statistical distributions with long tails (like the log-normal) or that arise as mixtures of distributions have values with digits that often satisfy Benford's Law (Hill 1995; Janvresse and de la Rue 2004). Under Benford's Law, the relative frequency of each second significant digit $j = 0, 1, 2, \ldots, 9$ in a set of numbers is given by $r_j = \sum_{k=1}^{9} \log_{10}(1 + (10k + j)^{-1})$ or $(r_0, \ldots, r_9) = (.120, .114, .109, .104, .100, .097, .093, .090, .088, .085)$. Benford's Law has been used to look for fraud in finance data (Cho and Gaines 2007).

In general the digits in vote counts do not follow Benford's Law (Carter Center 2005), but several examinations have found Benford's Law often approximately describes vote counts' second digits (e.g. Mebane 2006). Cantu and Saiegh (2011) find that Benford's Law approximately describes the first digits in some district-level election returns in some Argentine elections. I focus on precinct-level vote counts. It is best to think of precinct vote counts as following not Benford's Law but rather distributions in families of Benford-like distributions. Vote counts are mixtures of several distinct kinds of processes: some that determine the number of eligible voters in each precinct; some for how many eligible voters actually vote; some for which candidate each voter chooses; some for how the voter's choice is recorded. Such mixtures can produce numbers that follow Benford-like distributions but not Benford's Law (Rodriguez 2004; Grendar, Judge and Schechter 2007). While in previous work the following tests have been described as second-digit Benford's Law (2BL) tests, it is more precise to refer to second-digit Benford-like tests.

Tests for the second digits of vote counts come in two forms. One uses a Pearson chi-squared statistic tied to Benford's Law: $X^2_{2BL} = \sum_{j=0}^{9} (n_j - Nr_j)^2/(Nr_j)$, where N is the number of vote counts of 10 or greater (so there is a second digit), n_j is the number having second digit j and r_j is given by the Benford's Law formula. If the counts whose digits are being tested are statistically independent, then this statistic should be compared to the chi-squared distribution with nine degrees of freedom.

The second statistic, inspired by Grendar, Judge and Schechter (2007), is the mean of the second digits, denoted $\hat{j}$. If the counts' second-digits follow Benford's Law, then the value expected for the second-digit mean is $\bar{j} = \sum_{j=0}^{9} jr_j = 4.187$.

Election Examples

To illustrate the second-digit phenomena of interest, I consider precinct data from U.S. elections for president and the U.S. House and from German Bundestag elections. The U.S. elections are the presidential election of 2008 and the U.S. House elections of 1984.[2] For 2008 there are data for 40 states,[3] and for 1984 the data include every state except California. Data are not available for every precinct in some states. The German elections

[2]Data from 2008 were collected by the author. 1984 data come from the Record of American Democracy (ROAD) (King, Palmquist, Adams, Altman, Benoit, Gay, Lewis, Mayer and Reinhardt 1997) and from Office of the Clerk (2010).

[3]The states with data in 2008 are AK, AZ, AR, CA, CT, DC, DE, FL, GA, HI, ID, IL, IN, IA, KS, LA, ME, MD, MI, MN, MS, NH, NM, NY, NC, ND, OH, OK, PA, RI, SC, SD, TN, TX, VT, VA, WA, WV, WI, WY.

are the Bundestag elections of 2002, 2005 and 2009.[4]

Consider displays based on the votes recorded for president and for House elections, shown respectively in Figures 1 and 2. $\hat{j}$ is shown separately in four categories. Clockwise from the upper left in the display these are means for the Republican candidate in states where the Republican won, for the Republican candidate in states where the Democrat won, for the Democratic candidate in states where the Democrat won and for the Democratic candidate in states where the Republican won. In the display for the presidential election, states are placed along the x-axis at locations corresponding to the absolute margin between the Democratic and Republican candidates in each state.[5] Each plot shows a nonparametric regression curve[6] (Bowman and Azzalini 1997, 2003) that indicates how the mean of the second digit of the vote counts for the candidate in each category varies with the state absolute margin. Use $\hat{j}_x$ to denote this conditional mean. $\hat{j}_x$ is shown surrounded by 95 percent confidence bounds. In the display for the legislative election the x-axis contains the absolute margin in each legislative district.[7] The question in all the plots is whether $\bar{j}$, indicated by a horizontal dotted line in the plots, falls outside the confidence bounds. In such cases I say $\hat{j}_x$ differs significantly from $\bar{j}$.

*** Figures 1 and 2 about here ***

If the second digits followed the pattern expected given r_j derived from Benford's Law, then $\hat{j}_x$ would not differ significantly from $\bar{j}$, but evidently in Figure 1 it does differ in all states for the Democrat's votes where the Democrat won. The difference between $\hat{j}_x$ and $\bar{j}$ does not result simply from the fact that the Democrat got more votes in those places, because $\hat{j}_x$ mostly does not differ significantly from $\bar{j}$ for the Republican's votes in places where the Republican won. $\hat{j}_x$ is about 4.27 for most of the distribution for the Democrat's votes where the Democrat won.

[4]Data were obtained from Bundeswahlleiter (2010*b*,*a*, 2011*b*).

[5]In presidential races the absolute margin is the absolute difference between state vote proportions. Margins are based on state vote totals in Office of the Clerk (2010).

[6]Nonparametric regressions are computed using the `sm` package of **R** (R Development Core Team 2011).

[7]In legislative races the absolute margin is the difference between shares of the district two-party vote.

The second digits of 1984 U.S. House election vote counts also do not follow the pattern expected according to Benford's Law. In Figure 2, $\hat{j}_x > \bar{j}$ significantly over most of the distribution for Republican winners and over all of the distribution for Democratic winners. For losers of both parties $\hat{j}_x > \bar{j}$ significantly in close races but $\hat{j}_x < \bar{j}$ significantly in many races that are not so close. $\hat{j}_x$ ranges from a high of about 4.4 for some winners to a low of about 4.0 for some losers, with both highs and lows significantly different from $\bar{j}$.

Similar patterns occur for many other American elections, as I'll illustrate further below. Mebane (2008) noted a few departures from Benford's Law expectations using X^2_{2BL}, but as illustrated here much more extensive discrepancies become apparent when $\hat{j}_x$ is computed.

In the German case, consider displays based on *Erststimmen* and *Zweitstimmen* votes recorded in the Bundestag elections of 2002, 2005 and 2009. Each voter in these elections casts two votes. *Erststimmen* votes determine the winner of each *Wahlkreis* (district) through a plurality voting rule, and *Zweitstimmen* votes determine the overall share of the seats each party has in the Bundestag through proportional representation (PR) rules.[8] In Figure 3, the x-axis shows the margin between the first-place and second-place candidates in each *Wahlkreis* as a proportion of the valid ballots cast in the *Wahlkreis*, and the y-axis shows the difference between the number of *Zweitstimmen* and *Erststimmen* votes received in each *Wahlkreis* by SPD as a proportion of all ballots cast in the *Wahlkreis*.[9] I use $\mathfrak{M}_{12}$ to refer to the x-axis quantity and $\mathfrak{D}_{\mathrm{SPD}}$ to refer to the y-axis quantity. Previous work has used the difference between a party's *Erststimmen* and *Zweitstimmen* votes as an indicator of the number of strategic votes the party was receiving in its *Erststimmen* vote total (e.g. Cox 1996, 83; Bawn 1999). Shifts that are larger in absolute magnitude arguably indicate higher proportions of voters engaging in strategic vote switching, whether through "wasted

[8]To receive seats through the PR process, a party must receive more than five percent of the valid *Zweitstimmen* votes "in the electoral area" or win three *Wahlkreise* based on *Erststimmen* votes. PR outcomes as determined by the *Zweitstimmen* votes depend on the *Erststimmen* votes in other ways too complicated to explain here (Bundeswahlleiter 2011*a*, Section 6).

[9]SPD is the Social Democratic Party of Germany, CDU/CSU is the Christian Democratic Union/Christian Social Union and PDS/Linke is the Party of Democratic Socialism/The Left.

vote" reasoning, "threshold insurance" calculations or whatever (Herrmann and Pappi 2008; Shikano, Herrmann and Thurner 2009).

*** Figure 3 about here ***

The curved lines plotted in Figure 3 show contours of $\hat{\jmath}$ estimated using polling station vote counts' second digits—pooling data from all three elections—in a two-dimensional nonparametric regression[10] (Bowman and Azzalini 1997) with $\mathfrak{M}_{12}$ and $\mathfrak{D}_{\mathrm{SPD}}$ as regressors. The numbers shown in the figure along the lines report the values of $\hat{\jmath}$ along the referent contours. Use $\hat{\jmath}_{xy}$ to denote this conditional mean. Figure 3 is based on the digits of the SPD's vote counts from every *Wahlkreis* where SPD was the second-place party. Blank areas in the figure (such as the lower right corner) reflect combinations of $\mathfrak{M}_{12}$ and $\mathfrak{D}_{\mathrm{SPD}}$ values that do not occur in the data. Standard errors are not depicted in the figure, but a test of the two-dimensional regression model against a model in which both regressors have "no effect" shows that $\hat{\jmath}$ does vary significantly as a function of the two covariates.[11]

The second-digit means often are not those Benford's Law would imply. As $\mathfrak{M}_{12}$ increases, $\hat{\jmath}_{xy}$ tends to decrease, from $\hat{\jmath}_{xy} \approx 4.4$ for $\mathfrak{M}_{12} \approx 0$ down to $\hat{\jmath}_{xy} \approx 3.85$ for $\mathfrak{M}_{12} \approx .5$. Values of $\hat{\jmath}_{xy} \approx \bar{\jmath}$ occur only for $\mathfrak{M}_{12} \approx .2$. $\hat{\jmath}_{xy}$ also responds slightly to $\mathfrak{D}_{\mathrm{SPD}}$, particularly for negative values of $\mathfrak{D}_{\mathrm{SPD}}$.

Figure 4 plots the contours from a similar two-dimensional nonparametric regression based on the digits of the SPD's vote counts from every *Wahlkreis* where SPD was the first-place party. Now $\hat{\jmath}_{xy}$ varies significantly with $\mathfrak{D}_{\mathrm{SPD}}$, increasing from $\hat{\jmath}_{xy} \approx 4.3$ for $\mathfrak{D}_{\mathrm{SPD}} \approx 0$ to $\hat{\jmath}_{xy} \approx 4.55$ for $\mathfrak{D}_{\mathrm{SPD}} \approx -.12$ and $\mathfrak{M}_{12} \approx .3$. $\hat{\jmath}_{xy}$ also responds to $\mathfrak{M}_{12}$. $\hat{\jmath}_{xy} \approx 4.35$ for $\mathfrak{M}_{12} \approx 0$ and $\mathfrak{D}_{\mathrm{SPD}} \approx -.04$ and also for $\mathfrak{M}_{12} \approx .29$ and $\mathfrak{D}_{\mathrm{SPD}} \approx 0$. Nowhere in the figure is $\hat{\jmath}_{xy}$ near $\bar{\jmath}$.[12]

[10]Two-dimensional nonparametric regressions are computed using the `sm` package of **R** (R Development Core Team 2011). The `sm.regression()` function call uses `method=aicc`.

[11]The `sm.regression()` function call also includes the argument `model='no effect'` and reports "Test of no effect model: significance = 0," indicating a very small significance level.

[12]`sm.regression()` reports "Test of no effect model: significance = 0" for these contours, so it's reasonable to conclude that the differences of $\hat{\jmath}_{xy}$ from $\bar{\jmath}$ are significant.

*** Figure 4 about here ***

Similar patterns occur for other parties and in other elections in Germany. I'll illustrate this for some other parties in these same three elections further below.

I will show that for the most part these deviations from Benford's Law expectations are produced in presidential elections by strategic voting and in U.S. House elections and in German Bundestag elections by gerrymandering and strategic voting. Deviations from these expected patterns, and not simply deviations from the pattern implied by Benford's Law, may indicate coercion or conceivably some other kind of fraud.

Simulating Strategic Voting, Gerrymandering and Coercion

The simulation exercises in this section use only one kind of strategic behavior, namely "wasted-vote logic" by which some voters decide to vote not for their most preferred choice but instead for a lower ranked alternative in order to try to defeat an even lower ranked alternative that they believe is attracting more votes than their first choice is attracting (e.g. Cox 1994; Herrmann and Pappi 2008). This is because this kind of strategy can be implemented in a single electoral district, hence it is simpler than trying to simulate entire electoral or political systems would be. But the idea—which needs to be tested, of course—is that the effect that this particular kind of strategic behavior has on the digits of vote counts matches the effect that other kinds of strategic behavior have on vote counts. The way the results from the simulation match the real election data, as I will discuss subsequently, supports this idea.

I simulate a simple plurality election based on artificial preferences generated so that in the case of a preferentially balanced electorate nonstrategic votes approximately satisfy 2BL. For realism, to match in particular the findings of Mebane (2006), the first significant digits of the artificial votes do not satisfy Benford's Law. Then I simulate the effects of three kinds of manipulation: strategic voting according to wasted vote logic, where voters who most prefer a losing candidate switch their votes to one of the top two finishers;

coercion, where some voters vote for a candidate regardless of their preferences; and gerrymandering, where the balance of support is skewed between two leading candidates. The idea is to presume a baseline 2BL distribution, as that is often observed, and then to see what effect the manipulations have on the simulated precinct vote counts' second digits. The simulation is constructed as a Monte Carlo exercise, so results reflect the average from hypothetically rerunning the election under the same conditions many times.[13] In real data such repetitions do not occur, of course, but often the repeated sampling methodology is invoked to support studying observed statistics. We will see that many effects produced in simulation often appear in real data.

I simulate and then count votes by individuals in a set of 5,000 simulated precincts. Mebane (2006) and Mebane (2007) simulate precinct data that satisfy 2BL, and the approach taken here is prompted by ideas used in those simulations.

There are three simulations that represent variations of the same basic method. In the first the idea is to simulate precincts that contain individuals who have preferences for each of four candidates, preferences generated from a set of mixture distributions, where three of the candidates are on the ballot. One may think of precincts as having different concentrations of more or less intense partisans, even though of course there is no real political content to the numbers used in the simulation. In the second and third simulations there are respectively two and four candidates. The two-candidate case matches the situation in most American legislative elections: with only two candidates, wasted-vote logic has no effect. Preferences are skewed in these latter two simulations in a manner intended to represent gerrymandering or, more generally, any circumstance in an election that favors one candidate. Only one election is simulated at a time, so these simulations do not represent all features of gerrymander. Indeed, they represent any factor that produces systematic deviations from an electoral situtation that is balanced between two candidates.

Each precinct has a basic offset μ selected using a uniform distribution on the interval

[13] All simulation conditions are replicated 500 times.

$[-2-\kappa, 2-\kappa]$: $\mu \sim U(-2-\kappa, 2-\kappa)$, where the situation favors one of the candidates if $\kappa \neq 0$. This determines the average "partisanship" of voters in the precinct. Setting $\kappa = 0$ defines the balanced case. Gerrymanders are represented by setting $\kappa \neq 0$.

There is a randomly generated number of voters in each precinct who have similarly generated preferences. Let $n_0 \sim P(\mathcal{N})$ denote an initial value for the number of eligible voters in the precinct, based on the Poisson distribution with mean $\mathcal{N}$. In the current simulations, $\mathcal{N} = 1300$. The number of different types of eligible voters in the precinct is an integer $K \sim I(2, 25)$ chosen at random with probability 1/24 from the set $\{2, \ldots, 25\}$. The number of eligible voters of each type is a Poisson random variable $n_i \sim P(n_0/K)$, $i = 1, \ldots, K$. Hence the total number of eligible voters in the precinct is $\tilde{n} = \sum_{i=1}^{K} n_i$, and the proportion of eligible voters of type i is $\phi_i = n_i/\tilde{n}$.

Each voter has a preference for each candidate that depends on the voter's type. The proportions ϕ_i are used to distribute the preferences types around the precinct offset μ. The mean type set proportion is $K^{-1} \sum_{i=1}^{K} \phi_i = K^{-1}$. Using the normal distribution with mean zero and variance σ, denoted $N(0, \sigma)$, define $\nu_{ji} \sim N(0, \sigma\sqrt{10})$ and generate base values for the preferences for choice j of the eligible voters of type i by

$$\mu_{1i} = \mu + \left(\phi_i - K^{-1}\right) \nu_{1i} \tag{1a}$$

$$\mu_{2i} = -\mu_{1i} \tag{1b}$$

$$\mu_{3i} = -0.1 + \mu + \left(\phi_i - K^{-1}\right) \nu_{3i} \tag{1c}$$

$$\mu_{4i} = -0.2 + \mu + \left(\phi_i - K^{-1}\right) \nu_{4i} \tag{1d}$$

These preference values are used for the first simulation where there are four candidates. Each normal variate is selected independently for each j and i. Hence, for example, the base value of preferences for candidate 1 held by eligible voters of type i is distributed normally with mean μ and variance $10\sigma^2 \left(\phi_i - K^{-1}\right)^2$. The average base value for preferences among all eligible voters in the precinct is μ. If μ represents the basic "partisanship" of each precinct, then the $(\phi_i - K^{-1})\,\nu_{ji}$ values represent effects different issues, performance judgments, social positions, campaign strategies and whatnot have on sets of voters.

A more positive number indicates a candidate is more preferred. Candidates 1 and 2 come from opposite "parties," while candidates 3 and 4 are typically positioned with values that have the same sign as but are slightly more negative than the values assigned to candidate 1. This structure implies that when candidate 1 is preferred to candidate 2 (i.e., when $\mu_{1i} > 0 > \mu_{2i}$), candidates 3 or 4 have some chance to be the most preferred candidate, but when $\mu_{2i} > 0 > \mu_{1i}$ candidates 3 and 4 are much less likely to be preferred over candidate 2. One might think of this as a situation in which there are two candidates that are ideologically similar to candidate 1 but usually less preferred than candidate 1.

The second simulation, with two candidates, uses only base preferences (1a) and (1b).

The third simulation, with four candidates, uses preference definitions (1a) and (1b) and a slightly different definition for μ_{3i} and μ_{4i}: using uniform variates $u_{ji} \sim U(0,1)$,

$$\mu_{3i} = \begin{cases} -0.1 - \mu + (\phi_i - K^{-1})\,\nu_{3i}, & \text{if } u_{3i} \leq .5 \\ -0.1 + \mu + (\phi_i - K^{-1})\,\nu_{3i}, & \text{if } u_{3i} > .5 \end{cases} \tag{2a}$$

$$\mu_{4i} = \begin{cases} -1.5 - \mu + (\phi_i - K^{-1})\,\nu_{4i}, & \text{if } u_{4i} \leq .5 \\ -1.5 + \mu + (\phi_i - K^{-1})\,\nu_{4i}, & \text{if } u_{4i} > .5 \end{cases} \tag{2b}$$

where each u_{ji} is drawn independently. In contrast with the first simulation, here candidates 3 and 4 are symmetrically positioned relative the first two candidates: in this case the values of candidates 3 and 4 at random have the same sign as either candidate 1 or candidate 2 instead of almost always having the same sign as candidate 1.

To get preferences for individuals, I add a type 1 extreme value (Gumbel) distributed component to each individual's base preference value. Let $\epsilon_{jik} \sim G(0,1)$ denote a type 1 extreme value variate with mode 0 and spread 1. For candidate $j \in \{1,2,3,4\}$ or

$j \in \{1, 2\}$, each of the n_i individuals k of type i has preference $z_{jik} = \mu_{ji} + \epsilon_{jik}$, with the extreme value variates being chosen independently for each candidate and individual. Hence each voter in the simulation has the same error structure for its preference as is implied if μ_{ji} is observed up to a set of unknown linear parameters which are estimated using a simple multinomial logit choice model (McFadden 1973).

To define the baseline of votes that are cast in the absence of strategic considerations, I define variables that measure for each individual which candidate is the first choice. This is the candidate for which the individual has the highest preference value. An individual does not vote unless the preferred candidate's value exceeds a threshold v. This represents the idea that not every eligible voter votes, perhaps due to the cost of voting.

Simulation 1: Strategic Voting and Coercion

The first simulation sets $\kappa = 0$ and so focuses on strategic voting and coercion. In this simulation there are four candidates but only candidates 1, 2 and 3 actually run. All voters with a first-place preference for candidate 4 are coerced to vote for candidate 1 regardless of their other preferences. So for each candidate j, first-place indicator y_{jik} is defined to be 1 if all the inequalities in the corresponding one of the following definitions are true, zero otherwise:[14]

$$y_{1ik} = z_{1ik} > v \wedge z_{1ik} > z_{2ik} \wedge z_{1ik} > z_{3ik} \wedge z_{1ik} > z_{4ik} \tag{3a}$$

$$y_{2ik} = z_{2ik} > v \wedge z_{2ik} > z_{1ik} \wedge z_{2ik} > z_{3ik} \wedge z_{2ik} > z_{4ik} \tag{3b}$$

$$y_{3ik} = z_{3ik} > v \wedge z_{3ik} > z_{1ik} \wedge z_{3ik} > z_{2ik} \wedge z_{3ik} > z_{4ik} \tag{3c}$$

$$y_{4ik} = z_{4ik} > v \wedge z_{4ik} > z_{1ik} \wedge z_{4ik} > z_{2ik} \wedge z_{4ik} > z_{3ik} \tag{3d}$$

Either zero or one of the y_{jik} values for each individual k will be nonzero. The total of these would-be votes for each candidate j is the sum of the y_{jik} values: $y_j = \sum_i \sum_k y_{jik}$.

[14] $\wedge$ denotes logical 'and'.

The votes for candidates 1, 2 and 3 are subject to wasted-vote logic. I choose σ in equations (1a)–(1d) so that candidate 3 almost always has the smallest number of first-place finishes among candidates 1, 2 and 3. Hence some voters strategically abandon candidate 3 and vote for either candidate 1 or 2. The number of switches depends on both the relative valuations of the candidates and on whether the differences between candidates exceeds a threshold t: someone votes for their second-ranked candidate when their first-ranked candidate comes in last and the gaps between their choices are sufficiently large. Given that candidate 3 comes in last, the number of switched votes is

$$o_{312} = \sum_i \sum_k (z_{3ik} > v \wedge z_{3ik} > z_{1ik} + t \wedge z_{1ik} > z_{2ik} + t \wedge z_{3ik} > z_{4ik})$$

$$o_{321} = \sum_i \sum_k (z_{3ik} > v \wedge z_{3ik} > z_{2ik} + t \wedge z_{2ik} > z_{1ik} + t \wedge z_{3ik} > z_{4ik})$$

The votes for each candidate after the strategic switching to second-ranked candidates are

$$w_1 = y_1 + o_{312} \tag{4a}$$

$$w_2 = y_2 + o_{321} \tag{4b}$$

$$w_3 = y_3 - (o_{312} + o_{321}) \tag{4c}$$

Notice that if $t = 0$, then $w_3 = 0$ and candidate 3 receives no votes.

Because voters who place candidate 4 first are coerced to vote for candidate 1, the total of votes for candidate 1 is $\tilde{w}_1 = w_1 + y_4$.

Table 1 reports the mean over the replications of χ^2_{2BL}, $\hat{j}$, the standard error of $\hat{j}$ and the total number of would-be votes in y and votes in w and $\tilde{w}$.

*** Table 1 about here ***

The results show the pattern of second digits to be sensitive to all the manipulations implemented in the simulation.[15] First, looking at the statistics for the would-be votes y_j,

[15]The simulation results themselves are stable within a range of variation of the model conditions. Using $v = 2$ produced similar results, but using $v = 1.5$ produced departures from 2BL in y_2 that were detectable

χ^2_{2BL} for y_1 shows no significant departure from the 2BL pattern, while $\hat{j}$ is slightly more than two standard errors greater than $\bar{j}$: $4.29 - 2(.04) > \bar{j}$. This excess above $\bar{j}$ is caused by the presence of the two other candidates, 3 and 4, competing for first place when μ_{1i} is positive. This is evident upon contrasting the statistics for y_2. Except for the presence of candidates 3 and 4, the preferences underlying y_2 are symmetrically opposite those underlying y_1. Solely due to the symmetry in the preference distribution, the statistics should be the same. Yet while χ^2_{2BL} again shows no significant departure from the 2BL pattern, $\hat{j} = 4.15$ for y_2 is less than but not significantly different from $\bar{j}$.[16] Considered on their own, the counts of would-be votes for candidates 3 and 4 do not have significantly discrepant χ^2_{2BL} values but do have $\hat{j}$ values significantly greater than $\bar{j}$.

Once wasted-vote logic is used to shift some votes away from candidate 3 and to candidates 1 and 2, the distribution of second digits changes noticeably. For w_1 and w_2, χ^2_{2BL} shows no significant departure from 2BL, but $\hat{j}$ is significantly greater than $\bar{j}$. These mean statistics however remain significantly smaller than the value of 4.5 that would occur if the second digits were distributed with equal frequencies (meaning, if each occurred with probability 1/10). For w_3, χ^2_{2BL} is very significantly different from what 2BL would imply, and $\hat{j}$ is substantially less than $\bar{j}$. Of course, having set $t = 0$ would have reduced w_3 to exactly zero, but setting other small values for t produces similar results.[17]

Finally, the effect of coercion is evident in the statistics for $\tilde{w}_1$. χ^2_{2BL} is very significantly different from what 2BL would imply, and $\hat{j}$ is substantially less than $\bar{j}$. Notably $\hat{j}$ here is significantly greater than $\hat{j}$ for the candidate that was abandoned for strategic reasons. The vote counts differ for the candidates, however—candidate 1 has more than 35 times the vote of candidate 3—so there should be little possibility of confusion between candidates whose statistics differ because of these respective mechanisms.

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by χ^2_{2BL}. For $\mathcal{N} \in \{1200, 1400, 1500\}$, $\hat{j}$ for y_2 remains not significantly different from $\bar{j}$, so that the other statistics can be considered relevant. In these cases the statistics for the other vote totals behave as described in the text. For $\mathcal{N} \in \{800, 900, 1000, 1100\}$, $\hat{j}$ for y_2 differs significantly from $\bar{j}$.

[16]Here I use "significantly different" to refer to means that differ by more than two standard errors.

[17]I found similar results for all the statistics reported here for $t \in \{.5, .45, .4, .35, .3, .25, .2, .15, .1, .05, .025\}$.

Most important for the prospect of detecting coercion is that the statistics for $\tilde{w}_1$ differ substantially from those for w_1 or even y_1. In this case, with two candidates having balanced support except a third candidate is more similar to one of the two major candidates, the second digits of vote counts of winning candidates allow fraud done by coercion to be distinguished from either strategic or nonstrategic normal politics.

Distinguishing strategic from nonstrategic normal politics is a less of a sure bet. χ^2_{2BL} seems not to be useful for this purpose at all, but $\hat{j}$ does tell us something. The digit mean statistic for y_2 differs significantly from that for w_2, but the difference between $\hat{j}$ for y_1 and for w_1 falls a bit short of statistical significance. Increasing the number of precincts to 15,000 or more shrinks the standard error of the mean and consequently produces a significant difference. Hence we might surmise that with a sufficiently large number of precincts, $\hat{j}$ could distinguish between situations where a candidate has no ideologically (or more generally, preferentially) similar competition due to voters having strategically abandoned all such candidates from the situation where such candidates never existed. The latter case might arise, for instance, where elites or processes (say primaries or ballot access laws) act to keep the other candidates off the ballot and out of voters' considerations. A much larger number of precincts seem to be required to distinguish wasted-vote strategic voting from the situation where similar but less preferred candidates appear on the ballot in the absence of strategic voting. In both of these latter cases, significant deviations from 2BL in $\hat{j}$ can occur, but the mean appears to be slightly larger when there is strategic voting.

Simulation 2: Gerrymandering

The second simulation focuses on implications of gerrymandering. In this simulation there are two candidates. There is no strategic voting. The following inequalities determine votes:

$$y_{1ik} = z_{1ik} > v \wedge z_{1ik} > z_{2ik} \tag{5a}$$

$$y_{2ik} = z_{2ik} > v \wedge z_{2ik} > z_{1ik} \tag{5b}$$

The total votes for each candidate j is the sum of the y_{jik} values: $y_j = \sum_i \sum_k y_{jik}$.

In many cases, especially in plurality rule legislative elections that follow partisan primary elections, only two candidates are on the ballot, so strategic voting according to wasted-vote logic cannot happen. In such cases the two candidates often do not have balanced support, due to the drawing of legislative district lines and the effects of issues in the race, campaigns and other transient phenomena. I manipulate the value of κ to simulate the effect of such imbalances. I use $\kappa \in \{0, .2, .4, .6\}$ so that in unbalanced cases it is candidate 2 who has the advantage.

A frequent corollary of gerrymanders due to districting decisions is decreased voter turnout: voters who support a party that is disadvantaged in the drawing of district lines may not vote in the legislative race, in the belief, perhaps, that their favored candidate has no chance of winning. I modify the turnout threshold parameter in order to represent this possibility. The turnout threshold is specified to increase as a function of the ratio between the first-place preferences for candidate 1 and the first-place preferences for candidate 2. Define a logistic function of the ratio between votes for the two candidates as follows:

$$f_j = 2\,/(1 + \exp\left[b_j\left(1 - y_1/y_2\right)\right]) \tag{6}$$

If $y_1 = y_2$, then $f_j = 1$, but given turnout factor $b_j < 0$ then $y_1 < y_2$ implies $f_j > 1$. I use f_j

to modify the turnout threshold in the voting rule for candidate j. The modified votes are

$$y^*_{1ik} = z_{1ik} > f_1 v \wedge z_{1ik} > z_{2ik} \tag{7a}$$

$$y^*_{2ik} = z_{2ik} > f_2 v \wedge z_{2ik} > z_{1ik} \tag{7b}$$

As the gap between the votes for candidates 1 and 2 increases, an eligible voter who prefers candidate 1 has to have increasingly extreme preferences in order to motivate actually voting. Using y^*_{2ik} also allows voters for the advantaged party to vote less if they think the race will be lopsided. These votes total $y^*_j = \sum_i \sum_k y^*_{jik}$.

Some results from this simulation for $\kappa \in \{0, .2, .4, .6\}$ appear in Figure 5. The first row of the figure shows $\hat{j}_x$ computed from y^*_j and plotted against values of the turnout factor (which is x) in the case $b_1 = b_2$.[18] $\hat{j}_x$ almost never equals $\bar{j}$, the second-digit mean expected according to Benford's Law.[19] As the advantage to candidate 2 increases, the Monte Carlo mean of $\hat{j}_x$ increases and then decreases for candidate 1 but steadily decreases for candidate 2. At $\kappa = 0$ and $b_1 = b_2 = 0$, on average $\hat{j}_x$ is 4.20 for both candidates, but as the advantage increases through $\kappa = .2$ to $\kappa = .6$, even while holding $b_1 = b_2 = 0$, $\hat{j}_x$ for candidate 1 first increases to 4.32 then decreases to 4.03. In the same case for candidate 2, $\hat{j}_x$ decreases through 4.01 to 3.71. As turnout declines, $\hat{j}_x$ declines for candidate 1 but rises for candidate 2. Depending on turnout, $\hat{j}_x$ for candidate 2 may be either below or above $\bar{j}$.

*** Figure 5 about here ***

The second and third rows of Figure 5 provide some practical sense of the kinds of races the simulated conditions represent. The second row shows the margin of victory for candidate 2 over candidate 1, as a proportion. For each value of the advantage κ, the figure shows the relationship between the margin and the turnout decline factor applied three ways: when only votes for candidate 1 are affected ($b_2 = 0$); when only votes for candidate

[18] The simulation was actually run for all combinations of values $b_1, b_2 \in \{0, -.5, -1, -1.5, -2, -2.5, -3\}$. Figure 5 uses the values produced when $b_1 = b_2$. Other values are interpolated.

[19] The standard error of $\hat{j}_x$ is in the range .04 to .05.

2 are affected ($b_1 = 0$); and when both candidates are equally affected ($b_1 = b_2 < 0$). The margin increases as turnout for candidate 1 declines and decreases as turnout for candidate 2 declines, but it increases slightly as both candidates' turnout declines. The third row of the figure shows the proportion by which turnout decreases in each of the foregoing scenarios, taking the outcome when $\kappa = b_1 = b_2 = 0$ as a baseline. Turnout decreases most when both candidates are affected and least when only candidate 1 is affected.

Figure 6 emphasizes the nonlinear effect candidate advantage has on $\hat{j}_x$ and how that effect depends on voter turnout. Each plot in the figure relates $\hat{j}_x$ to κ as the candidate advantage κ increases (now κ is x).[20] Plots are shown for turnout factors $b_1 = b_2 \in \{0, -2\}$. When $b_1 = b_2 = 0$, then as candidate 2's advantage increases a peak in $\hat{j}_x$ is evident for candidate 1 at $\kappa = .2$ but $\hat{j}_x$ for candidate 2 decreases steadily. But when $b_1 = b_2 = -2$, the peak for candidate 1 in $\hat{j}_x$ occurs at a slightly smaller value of κ, and $\hat{j}_x$ for candidate 2 increases—with a peak at $\kappa = .4$—before it decreases.

*** Figure 6 about here ***

Simulation 3: Gerrymandering, Strategic Voting and Coercion

The third simulation features gerrymandering, strategic voting and coercion. In this simulation there are again four candidates but only candidates 1, 2 and 3 actually run. Votes in this simulation reflect a combination of the logics used in the first two simulations. All voters with a first-place preference for candidate 4 are coerced to vote for either candidate 1 or candidate 2 regardless of their other preferences. First-place indicator variables y_{jik} are defined by (3a)–(3d). Using f_j as defined in (6), vote thresholds for candidates 1 and 2 are modified according to (7). The threshold for candidate 4 is also modified by f_1: $y^*_{4ik} = z_{4ik} > f_1 v \wedge z_{4ik} > z_{1ik} \wedge z_{4ik} > z_{2ik} \wedge z_{4ik} > z_{3ik}$. The number of

[20]Figure 6 uses the values produced when $\kappa \in \{0, .05, .1, .15, .2, .25, .3, .35, .4, .45, .5, .55, .6, .65, .7, .75, .8, .85\}$. Other values are interpolated.

switched votes is now

$$o^*_{312} = \sum_i \sum_k (z_{3ik} > f_2 v \wedge z_{3ik} > z_{1ik} + t \wedge z_{1ik} > z_{2ik} + t \wedge z_{3ik} > z_{4ik})$$
$$o^*_{321} = \sum_i \sum_k (z_{3ik} > f_1 v \wedge z_{3ik} > z_{2ik} + t \wedge z_{2ik} > z_{1ik} + t \wedge z_{3ik} > z_{4ik})$$

The votes for candidates 1 and 2 after strategic switching to second-ranked candidates are

$$w^*_1 = y^*_1 + o^*_{312}$$
$$w^*_2 = y^*_2 + o^*_{321}$$

For the case $b_1 = b_2 = 0$, the main difference between the first simulation and the third is the symmetry in the generation of preferences for candidates 3 and 4 in the third simulation. Here these candidates are as likely to attract preferences with same sign as candidate 1 as they are candidate 2. While the rule for strategic vote switching according to wasted-vote logic is the same in both simulations, I treat the first-place preferences for candidate 4 differently. Now I consider cases where all first-place finishes for candidate 4 are assigned to either candidate 1 or candidate 2: $\tilde{y}^*_j = y^*_j + y^*_4$ or $\tilde{w}^*_j = w^*_j + y^*_4$ for $j \in \{1, 2\}$. In these cases, those who most prefer candidate 4 are coerced.

Figure 7 shows the combined effects of strategic voting, gerrymandering and coercion in this case of symmetric third-party preferences, ignoring turnout effects. The figure plots $\hat{\jmath}_x$ for candidates 1 and 2 against κ in four scenarios, two without strategic voting and two with.[21] The (a) and (b) plots show results for votes with no coercion, respectively y^*_j and w^*_j. The (c) and (d) plots show results for votes including coerced votes: $\tilde{y}^*_j$ and $\tilde{w}^*_j$. In almost all cases, the effect of strategic voting is to reduce $\hat{\jmath}_x$: with strategic voting, $\hat{\jmath}_x$ never exceeds $\bar{\jmath}$ whereas without strategic voting it sometimes does. Without strategic voting, $\hat{\jmath}_x$ is not significantly different from $\bar{\jmath}$ for low levels of candidate advantage.[22] Adding coerced

[21]Figure 7 uses the values produced when $\kappa \in \{0, .05, .1, .15, .2, .4, .5, .6\}$. Other values are interpolated.
[22]The standard error for $\hat{\jmath}_x$ is usually about .05.

votes increases $\hat{j}_x$ for candidate 1 but has a negligible effect on $\hat{j}_x$ for candidate 2. The patterns seen with turnout effects enacted are similar to those seen in Figure 7.[23]

*** Figure 7 about here ***

Simulation Overview

The simulations suggest the second-digit means of precinct vote counts are sensitive to many kinds of manipulation. The second simulation shows that even without any kind of election fraud at all, normal politics in the form of gerrymandering can produce an array of distinctive patterns. The first and third simulations show that strategic voting can do so as well. When strategic voting is asymmetric, $\hat{j}$ can distinguish strategic voting from coercion much more effectively than when strategic voting is symmetric.

The idea of symmetry in strategic voting is relevent to the question of distinguishing two kinds of strategic voting. If one thinks in terms of a one-dimensional spatial model of politics, then one will probably observe that in presidential elections there are fringe parties on both the left and right, so it is not easy to see that occasions for strongly asymmetric wasted-vote actions, as in the first simulation, will routinely occur. But in the strategic theory of party balancing of Alesina and Rosenthal (1995), strategic switchers all go one way—only one party's presidential candidate and House candidates of the opposite party gain strategic votes—and substantial asymmetry emerges in the empirical estimates of Mebane (2000, 53). In terms of the pattern the simulation predicts for $\hat{j}$, in the case of asymmetric strategic switching as in the first simulation, strategic voting implies $\hat{j} > \bar{j}$ while the symmetric case of the third simulation implies $\hat{j} < \bar{j}$. Evidence of asymmetry in strategic voting in the U.S. election data would be a result of strategic party balancing.

The margin in a race is an almost always measurable covariate with respect to which to array $\hat{j}_x$ values. If the second digits of precinct votes counts are available, then probably so are the counts themselves, so margins should be feasible to compute. Exceptions will occur

[23]Specifically, the plots are very similar when $b_1 = b_2 = -2$.

when not all precincts are available and neither are constituency totals.

Turnout also evidently can be important in determining $\hat{j}$, but it is a fuzzier concept and one more difficult to measure than the margin of victory. The baseline of eligible voters can be tricky to define and hard to obtain (McDonald and Popkin 2001). The fuzziness relates to rolloff (Wattenberg, McAllister and Salvanto 2000; Herron and Sekhon 2005): is mere attendance at the polls sufficient or must one vote for a particular office?

In connection with U.S. House elections voter turnout is relevant for evaluating whether a principal feature of the simulation of gerrymander is appropriate. The simulations particularly investigate the effects of turnout declining as a function of candidate advantage. Does it so decline? Ferejohn and Fiorina (1975) present a skeptical argument based on electors' expectations that an election will be "close" and survey data. Cox and Munger (1989) and Berch (1989) show that turnout does decline when the election is close, although the phenomenon depends on elite behavior.

The simulations, while perhaps complicated, are not particularly realistic. Precinct sizes, for instance, do not generally follow a mixed Poisson distribution.[24] Other features of the simulations also are admittedly artificial. The least one can say is that real data represent mixtures that are much more complicated and irregular than the simulations. Rather than attempt to make the simulations much more realistic, I turn instead to their qualitative correspondence with real data from some actual elections.

German Bundestag Elections, 2002–2009

Consider the pattern previously observed for the German Bundestag elections of 2002, 2005 and 2009, in Figure 3, for the second digits of polling station counts of votes cast for the SPD in *Wahlkreise* where the SPD is the party that received the second highest number of votes in the *Wahlkreis*. The variations in the digits can explained by gerrymandering, with rolloff, and strategic voting: the pattern of $\hat{j}_{xy}$ values in the figure match the patterns

[24]Nor do precinct sizes follow a negative binomial distribution as was used in the "calibration" effort of Mebane (2007).

observed in the first and second simulations.

To see the gerrymandering point, compare $\hat{j}_{xy}$ as it varies with $\mathfrak{M}_{12}$ in Figure 3, for $\mathfrak{D}_{\text{SPD}}$ values near zero, to the line ($\hat{j}_x$) that shows the pattern of digits for the first (disadvantaged) candidate as "2d candidate advantage" increases in Figure 6—in particular the curve in the "both candidates turnout decline" graph. The "2d candidate advantage" is exactly the margin expected between the first- and second-place candidates and so is comparable to $\mathfrak{M}_{12}$ shown in Figure 3, although of course the simulation includes exactly two candidates while in the German elections there are many more than two parties with candidates in each *Wahlkreis*. In Figure 3 there are no data with $\mathfrak{D}_{\text{SPD}} = 0$ for $\mathfrak{M}_{12}$ less than about .04, so the parts of Figure 6 corresponding to the very lowest values of "2d candidate advantage" lack matches in Figure 3. But for the parts of Figure 6 where $\mathfrak{M}_{12}$ equals "2d candidate advantage," the curve for the disadvantaged ("first") candidate in Figure 6 has a pattern of $\hat{j}_x$ values declining as "2d candidate advantage" increases just as Figure 3 has $\hat{j}_{xy}$ declining as $\mathfrak{M}_{12}$ increases. The values of $\hat{j}_x$ tend to be greater than values of $\hat{j}_{xy}$ for corresponding values of $\mathfrak{M}_{12}$. For instance, $\hat{j}_{xy} \approx 3.95$ and $\hat{j}_x \approx 4.05$ when $\mathfrak{M}_{12} = .4$, $\hat{j}_{xy} \approx 4.15$ and $\hat{j}_x \approx 4.3$ when $\mathfrak{M}_{12} = .2$ and $\hat{j}_{xy} \approx 4.25$ and $\hat{j}_x$ is slightly greater than 4.3 when $\mathfrak{M}_{12} = .1$. Still the qualitative correspondence is remarkable.[25] Looking at the values of $\hat{j}_{xy}$ only where $\mathfrak{D}_{\text{SPD}} \approx 0$ is reasonable because the simulation that produces Figure 6 includes no strategic voting, so if $\mathfrak{D}_{\text{SPD}}$ does roughly correspond to the amount of strategically switched votes, using places where $\mathfrak{D}_{\text{SPD}} \approx 0$ constrains the empirical analysis to places where strategic switching is minimal.

The correspondence between the simulation and what actually happens in German elections is not unique to SPD. Figure 8 shows $\hat{j}_{xy}$ estimated for the second digits of polling station counts of votes cast for CDU/CSU in *Wahlkreise* where CDU/CSU is the party that received the second greatest number of votes in the *Wahlkreis*. Now the y-axis is

[25] Indeed, the simulation was completed (Mebane 2010*a*) two years before the German data were collected and analyzed. The first attempt to apply the results of the simulation to German election data (from the 2009 election) was Gatof (2010). Using other values for b_1 and b_2 in (6) in simulation 2 brings $\hat{j}_x$ closer to $\hat{j}_{xy}$ for the corresponding Margin value.

defined in terms of *Erststimmen* and *Zweitstimmen* votes for CDU/CSU, and the measure is denoted by $\mathfrak{D}_{\text{CDU}}$. As in Figure 3, $\hat{j}_{xy}$ declines as $\mathfrak{M}_{12}$ increases. The values of $\hat{j}_{xy}$ for $\mathfrak{D}_{\text{CDU}} \approx 0$ in Figure 8 are similar to those in Figure 3. To the extent partisan imbalances among voters are the reason for the margins in these *Wahlkreise*, the second simulation suggests that those gerrymanders considerably explain these patterns in the second digits of the vote counts.

*** Figure 8 about here ***

The relevance of the "both candidates turnout decline" graph in Figure 6 to these German election data partly depends on there being a pattern of decline in turnout with increasing election margins in the German elections. Without such a decline, the simulation results most relevant to the German data would be those reported in the "no turnout decline" graph in Figure 6. That graph qualitatively resembles $\hat{j}_{xy}$ in the real data in that the simulated $\hat{j}_x$ eventually declines as the margin increases, but quantitatively $\hat{j}_x$ when there is "no turnout decline" is much farther from $\hat{j}_{xy}$ than $\hat{j}_x$ in the situation where "both candidates turnout decline." Stiefbold (1965) writes that the invalid vote in Germany "expresses a variety of political discontent" (Stiefbold 1965, 392), but the concept of turnout that relates to the simulation must take into account those eligible voters who do not cast votes at all. For the 2009 election only, Figure 9 plots $\mathfrak{M}_{12}$ against turnout as measured by Bundeswahlleiter (2012). Turnout generally declines as $\mathfrak{M}_{12}$ increases, although the pattern of decline is weakest in the subset of *Wahlkreise* where CDU/CSU won. The patterns of decline qualitatively validate the second simulation, although the implementation of "rolloff" in (6) is not a correct model in quantitative detail for Germany.

*** Figure 9 about here ***

The second simulation seems to match the real election data when $\mathfrak{D}_{\text{SPD}} \approx 0$ and $\mathfrak{D}_{\text{CDU}} \approx 0$, but what happens with larger magnitudes of $\mathfrak{D}_{\text{SPD}}$ and $\mathfrak{D}_{\text{CDU}}$—when the $\mathfrak{D}_k$ measures suggest there is a subtantial amount of strategic vote switching? I return to this question in relation to the *Wahlkreise* where SPD or CDU/CSU finished second in a moment, but first it is convenient to consider those places where SPD won. These are the *Wahlkreise* for which $\hat{j}_{xy}$ values are estimated in Figure 4.

The immediate thing to notice in Figure 4 is that the value of $\hat{j}_{xy}$ increases as $\mathfrak{D}_{\text{SPD}}$ becomes more negative. For $\mathfrak{M}_{12} \approx 0$, we have $\hat{j}_{xy} = 4.3$ for $\mathfrak{D}_{\text{SPD}}$ just above zero, $\hat{j}_{xy} = 4.35$ for $\mathfrak{D}_{\text{SPD}} \approx -.04$ and $\hat{j}_{xy} = 4.4$ for $\mathfrak{D}_{\text{SPD}} \approx -.1$. The $\hat{j}_{xy}$ values rise to match the $\hat{j}$ values observed in the first simulation when a party receives strategically switched votes. Indeed, $\hat{j}_{xy} = 4.4$ is larger than the value of $\hat{j} = 4.35$ that occurred with strategic switching in that asymmetric case, as reported in Table 1. The smallest value for $\hat{j}_{xy}$ in Figure 4, $\hat{j}_{xy} = 4.3$, matches the value reported in Table 1 for the digits in the sincere vote counts (y_1) for the party that has an ideologically similar party competing against it in the same district.[26] Higher values of $\hat{j}_{xy}$ occur for more negative values of $\mathfrak{D}_{\text{SPD}}$ and as $\mathfrak{M}_{12}$ rises. The highest value of $\hat{j}_{xy}$, namely, $\hat{j}_{xy} = 4.55$, occurs in the figure for $\mathfrak{D}_{\text{SPD}} \approx -.12$ and $\mathfrak{M}_{12} \approx .3$. Values of $\hat{j}$ this large do not occur in the first simulation, but that simulation also does not produce $\mathfrak{M}_{12}$ as large as occurs in Figure 4.

The value of $\hat{j}_{xy}$ seems strongly related to the amount of strategic vote switching, but thinking about the "wasted vote logic" that is likely the reason for the vote switching (Herrmann and Pappi 2008) suggests that the margin variable being used in the analysis is not the most appropriate one. The key quantity in such strategic voting is not the difference between the top two finishers but rather the differences between each of those parties and the party that comes in third. With "Duvergerian" equilibria (Cox 1994) in single-member districts, two parties get almost all the votes and votes for all other parties are reduced to negligible amounts. So margins defined relative to the third-place finisher are arguably more indicative of strategic activity than the margin between the top two finishers, even if the top-two margin may be most relevant when thinking about turnout declines caused by gerrymandering.[27]

[26]The *Wahlkreise* that have $\hat{j}_{xy} = 4.3$ in the pooled 2002, 2005 and 2009 data are in the eastern German *Länder* Mecklenburg-Vorpommern, Saxony-Anhalt and Thuringia.

[27]The 0-1 hypothesis introduced by Cox (1994) involves the margin between the second- and third-place candidates.

Figure 10 shows $\hat{j}_{xy}$ estimates using the digits of the SPD's vote counts from *Wahlkreise* where SPD was the first-place party, as in Figure 4, except using $\mathfrak{M}_{13}$, the margin between the first- and third-place parties in each *Wahlkreis*. The contours in Figure 10 are somewhat more horizontal than in Figure 4, so that $\hat{j}_{xy}$ varies more with $\mathfrak{D}_{\text{SPD}}$ than it does with $\mathfrak{M}_{13}$. The minimum value of $\hat{j}_{xy}$ is the 4.28 for $\mathfrak{D}_{\text{SPD}} \approx .035$, very similar to the minimum value with $\mathfrak{M}_{12}$. $\hat{j}_{xy}$ now has a maximum of $\hat{j}_{xy} = 4.46$, less than in Figure 4. The maximum of $\hat{j}_{xy}$ in Figure 10 corresponds to the most negative value of $\mathfrak{D}_{\text{SPD}}$, however, so $\hat{j}_{xy}$ might be said more strictly to increase with the amount of strategic vote switching.

*** Figure 10 about here ***

Using $\mathfrak{M}_{23}$, the margin between the second- and third-place parties, to estimate $\hat{j}_{xy}$ for the second digits of votes cast for SPD in *Wahlkreise* where SPD is the second-place party produces the contours shown in Figure 11. Compared to Figure 3, which I interpreted in terms of gerrymandering, the $\hat{j}_{xy}$ contours in Figure 11 are more horizontal, at least for $\mathfrak{D}_{\text{SPD}} < 0$, and for $\mathfrak{D}_{\text{SPD}} < 0$ $\hat{j}_{xy}$ increases as $\mathfrak{D}_{\text{SPD}}$ decreases and $\mathfrak{M}_{23}$ increases. The maximum value, now $\hat{j}_{xy} = 4.5$, occurs when both $\mathfrak{D}_{\text{SPD}} \approx -.125$, near the most negative value, and $\mathfrak{M}_{23}$ is near its maximum value.[28] When $\mathfrak{D}_{\text{SPD}} < 0$ indicates that many strategically switched votes are being added to the SPD's totals, so do the second digits of the votes through $\hat{j}_{xy}$.

When $\mathfrak{D}_{\text{SPD}} > 0$, presumably strategically switched votes are being subtracted from the SPD's *Erststimmen* totals. $\hat{j}_{xy}$ tends to get smaller as $\mathfrak{D}_{\text{SPD}}$ increases above zero. The minimum value $\hat{j}_{xy} = 3.9$ occurs for the most positive value $\mathfrak{D}_{\text{SPD}} \approx .15$. The five most positive values of $\mathfrak{D}_{\text{SPD}}$ in Figure 11—all the values where $\mathfrak{D}_{\text{SPD}} > .05$—occur in Berlin in *Wahlkreise* where Die Linke won in 2009. The $\mathfrak{D}_{\text{SPD}}$ values therefore suggest some voters who sincerely prefer SPD are switching strategically to vote for Die Linke in those *Wahlkreise*. This scenario where a large party is strategically abandoned by a substantial but still small proportion of voters is not represented in the simulations, although

[28]Interestingly, the *Wahlkreise* that have large $\mathfrak{M}_{23}$ in Figure 11 have $\mathfrak{M}_{12} \approx 0$ in Figure 3.

numerically the values of $\hat{j}_{xy}$ are close to the values of $\hat{j}_x$ for the disadvantaged candidate in the third simulation where there is both strategic voting and turnout decline due to gerrymandering (see plot (b) in Figure 7). It seems that the limited strategic abandonment, which $\mathfrak{D}_{\mathrm{SPD}} > 0$ indicates is occurring even though the vote for SPD is not being reduced to negligible amounts, has an effect on the vote counts' second digits similar to the effect turnout decline has on vote counts' digits in the simulation.

Using $\mathfrak{D}_{\mathrm{CDU}}$ and $\mathfrak{M}_{23}$ with the second digits of votes cast for CDU/CSU in *Wahlkreise* where CDU/CSU is the second-place party produces nearly vertical contours in $\hat{j}_{xy}$. See Figure 12. Since the contours of $\hat{j}_{xy}$ are not perfectly vertical, $\mathfrak{D}_{\mathrm{SPD}}$ affects $\hat{j}_{xy}$ to some extent. So strategic vote switching that adds to the votes for CDU/CSU in places where CDU/CSU finished second does affect $\hat{j}_{xy}$ when the amount of vote switching is large enough. $\hat{j}_{xy}$ reaches 4.35 only for $\mathfrak{M}_{23}$ greater than about .22. As was true for SPD in (recall Figures 11 and 3), *Wahlkreise* that have large $\mathfrak{M}_{23}$ in Figure 12 have $\mathfrak{M}_{12} \approx 0$ in Figure 8.

*** Figure 12 about here ***

The $\hat{j}_{xy}$ contours when $\hat{j}_{xy}$ is estimated using $\mathfrak{M}_{13}$ and the digits of the CDU/CSU vote counts from *Wahlkreise* where CDU/CSU is the first-place party—Figure 13—are similar to those observed in Figure 10 when SPD is the first-place party. Looking from left to right, the contours start off horizontal, meaning the $\hat{j}_{xy}$ is solely a function of $\mathfrak{D}_{\mathrm{CDU}}$, then they tilt upward slightly, indicating that $\mathfrak{M}_{13}$ also affects $\hat{j}_{xy}$. As was the case in Figure 10, the maximum $\hat{j}_{xy} = 4.5$ in Figure 13 corresponds to the most negative value of $\mathfrak{D}_{\mathrm{SPD}}$, so $\hat{j}_{xy}$ is higher when the amount of strategic vote switching to CDU/CSU is higher. Very positive values of $\mathfrak{D}_{\mathrm{CDU}}$ do not occur.

*** Figure 13 about here ***

Herrmann and Pappi (2008) argue that PDS/Linke should be treated as a "large" party.[29] If $\hat{j}_{xy}$ is estimated as a function of $\mathfrak{M}_{13}$ and $\mathfrak{D}_{\mathrm{PDS}}$ using the *Wahlkreise* where

[29]Regarding PDS, "the clear-cut distinction between large parties and small parties, which is commonly assumed in the literature does not readily extend to East German constituencies" (Herrmann and Pappi

PDS/Linke won, the number of such *Wahlkreise* is too small to show that $\hat{\jmath}_{xy}$ varies significantly with the two covariates.[30] Nonetheless, the patterns in $\hat{\jmath}_{xy}$ for PDS/Linke are similar to those for SPD and CDU/CSU, so it is likely they are not terribly misleading. Figure 14 shows that $\hat{\jmath}_{xy}$ ranges from $\hat{\jmath}_{xy} = 4.26$ for $\mathfrak{D}_{\text{PDS}} \approx -.015$ up to $\hat{\jmath}_{xy} = 4.52$ for $\mathfrak{D}_{\text{PDS}} \approx -.08$. The *Wahlkreise* that have $\hat{\jmath}_{xy} \approx 4.5$ in Figure 14 include the five Berlin *Wahlkreise* that had relative small values of $\hat{\jmath}_{xy}$ and positive values of $\mathfrak{D}_{\text{SPD}}$ in Figure 11. The results for the Berlin *Wahlkreise* in Figures 11 and 14 together document the story about some voters strategically switching their votes from SPD to Die Linke in the 2009 election—if, that is, $\hat{\jmath}_{xy}$ relates to strategic voting as I have been arguing it does.

*** Figure 14 about here ***

Estimating $\hat{\jmath}_{xy}$ with slightly different covariates suggests that the point that PDS/Linke should be treated as a "large" party among East Germans does not imply that, from a strategic point of view, PDS/Linke is exactly analogous to SPD and CDU/CSU. I define $\mathfrak{D}_{\text{Other}}$ by treating all parties other than SPD, CDU/CSU and PSD/Linke as a single alternative ("Other"). Such a measure sums all strategic vote switching activity involving a small party, without specifying which large party the switching may relate to.[31] The top-left plot in Figure 15 shows that $\hat{\jmath}_{xy}$ for PDS/Linke is not at all a function of $\mathfrak{D}_{\text{Other}}$—the contour lines for $\hat{\jmath}_{xy}$ are vertical—so the strategic vote switching that occurs to support PDS/Linke in the *Erststimmen* does not involve substantial support from those who sincerely support small parties. In contrast, plots of $\hat{\jmath}_{xy}$ as a function of $\mathfrak{M}_{13}$ and $\mathfrak{D}_{\text{Other}}$ for SPD and CDU/CSU show horizontal or diagonal contours: strategic vote switching that occurs to support these parties' *Erststimmen* outcomes do involve small parties. To say this is not say anything novel: "Red-Green" (SPD-Green) coalitions and coalitions between the CDU/CSU and FDP[32] are well known (Gschwend and Pappi 2004;

2008, 233).

[30]`sm.regression()` reports "Test of no effect model: significance = 0.118" for these contours.

[31]To measure voter switching from small parties to either SPD or CDU/CSU under wasted vote logic, "one can use the difference between the sum of second and first votes of a small party as a proxy" (Pappi and Thurner 2002, 211).

[32]FDP is Free Democratic Party.

Spiegel Online 2012*b*).[33] The bottom-right plot in Figure 15 shows $\hat{j}_{xy}$ for the CDU/CSU vote count digits estimated using $\mathfrak{D}_{\mathrm{FDP}}$, which is constructed using the *Erststimmen* and *Zweitstimmen* votes for FDP. More *Erststimmen* votes being strategically switched away from FDP, as measured by $\mathfrak{D}_{\mathrm{FDP}} > 0$, go with an increase in $\hat{j}_{xy}$. $\hat{j}_{xy}$ rises from $\hat{j}_{xy} = 4.2$ for $\mathfrak{D}_{\mathrm{FDP}} \approx .01$ to $\hat{j}_{xy} = 4.42$ for $\mathfrak{D}_{\mathrm{FDP}} \approx .1$.

*** Figure 15 about here ***

The distinctive pattern in Figure 15 for PDS/Linke resonates with the claim that eastern German voters are more ideological than western German voters (Rohrschneider, Schmitt-Beck and Jung 2012), or at least distinctively ideological, since PDS/Linke takes policy positions often represented as occurring on a distinctive ideological dimension (Schofield and Sened 2006, 201–202; Schofield 2008, 154–155). It's not that eastern German voters are less strategic than western German voters are, but their preferences—for "socialism," according to Rohrschneider, Schmitt-Beck and Jung (2012, 24)—lead them to treat their *Zweitstimmen* votes in a distinctive manner. The strategies the eastern German voters are using seem peculiar to the mixed electoral system that exists in Germany. The hostility between the SPD and PDS/Linke is well known and longstanding (e.g. Spiegel Online 2012*a*), and simple "wasted vote logic" cannot explain why voters would switch from their sincere first-place preference to another party when their most preferred party finishes in second place. But, from Figures 11 and 14, that's apparently what happened in Berlin in 2009. This may be an instance where considering "activist valence" (Schofield and Sened 2006) is particularly important (e.g. Connolly 2010).

Elections in the United States, 1980s and Late 2000s

The most comprehensive form of strategic behavior that affects American elections is a consequence of the large-scale equilibrium that connects votes in presidential and legislative

[33] During the two weeks preceding the 2002, 2005 and 2009 elections *Der Spiegel* included the following aggregate numbers of stories regarding each named coalition: CDU/CSU-FDP, 22; SPD-FDP, 7; SPD-FDP-Grüne, 7; SPD-Grüne, 28; SPD-Grüne-Linke, 5; and SPD-Linke, 4 (Gschwend and Pappi 2004 and compilations performed for the author.)

elections, an equilibrium that is explained in the strategic coordination theory developed by Alesina and Rosenthal (1989, 1995, 1996). This theory has empirical implications that have been confirmed for elections and macroeconomic outcomes (e.g. Alesina, Londregan and Rosenthal 1993; Alesina and Rosenthal 1995), as well as for individual voting behavior in presidential and midterm House elections (Mebane 2000; Mebane and Sekhon 2002).

Voting equilibria in Alesina and Rosenthal's theory have a key property that relates to split-ticket voting in presidential elections: "the voters who split their ticket are always a fraction of those voting for the presidential candidate more likely to win" (Alesina and Rosenthal 1996, 1334). Given relatively homogeneous party platforms in the presidential race and in different legislative districts, this suggests that we should observe strategically switched votes being added to the vote counts of legislative candidates only of the party opposite that of the winning presidential candidate. But votes for the president are also affected by strategic considerations, since the "presidential cutpoint" in the theory is in equilibrium with the "legislative cutpoint": if as is usual the parties have divergent policy platforms (Alesina and Rosenthal 2000), then the more likely voters are to support the president of one party, the more voters give support to legislators of the other party, and vice versa (Alesina and Rosenthal 1995, 92–101). So one may say that the winning presidential candidate has gained more strategic votes than the losing candidate, even though it's tricky to express this precisely in terms of "strategically switched" votes.

This equilibrium relationship is the basis for expecting that the digits in vote counts from American elections for president and legislature will show evidence of asymmetric strategic behavior: signs of strategic votes being added to the presidential candidate of one party—Democrat or Republican—should appear in conjunction with signs of strategic votes being added to the votes of legislators of the other party.

A complication is that the version of Alesina and Rosenthal's model I've been referring to assumes each party is homogeneous, or nearly so, in its policy positions. But by 2008 tension between the Tea Party[34] and "mainstream" Republicans became clear. Alesina and Rosenthal (1995, 124) note that in some cases heterogeneous parties can produce "equilibrium policies [that] are exterior to the president's ideal policy." While it's not clear what if any configuration of party position values faithfully represents the emergence of the Tea Party, it is possible that such a configuration may imply votes are being strategically switched to legislative candidates of the same party as the president, unlike what happens with homogeneous parties. Such behavior should be apparent in vote counts' digits.

In midterm elections Alesina and Rosenthal's model implies all voters act sincerely so that there should be no evidence then of strategic vote switching behavior. In such elections we expect to see variation in vote counts' digits due only to gerrymandering and its consequences. Party heterogeneity may have an unexpected effect on vote counts' digits, as it may do in presidential elections.

Fiorina (1992) suggests that party balancing also has implications for state elections. There may be ties between state-level elections and the presidential election. Fiorina (1992) suggests that state-level elections may balance presidential elections: if one party gains support at the federal level, the other party may gain more support at the state level. Fiorina's argument for this does not depend on strategic behavior, although it is easy to imagine—at least in vague terms—how a theory analogous to Alesina and Rosenthal (1995) might be developed.

I consider precinct data from several kinds of elections conducted in the United States of America during the 1980s, 1990 and the 2000s.[35] I have vote totals reported for both federal and state offices. For the 1980s and 1990 the data include every state except California. For the other years data were obtained for most but not all states (including DC): 33 states in 2006 and 41 states in 2008.[36] Data are not available for every precinct in

[34] T.E.A.: Taxed Enough Already (Engstrom 2009).

[35] The 1980s and 1990 precinct data come from ROAD (King et al. 1997). Data from 2006 and 2008 were collected by the author. U.S. House and president margin data are computed from Office of the Clerk (2010)

[36] The states with data in 2006 are AL, AK, AZ, AR, CA, DE, FL, GA, HI, ID, IA, KS, LA, ME, MD, MI, MN, MS, NE, NH, NY, NC, ND, OH, PA, RI, SC, TN, TX, VT, VA, WI, WY.

some states.

First consider how the simulations bear on one of the real data examples introduced above. In Figure 2, $\hat{j}_x$ has values of about 4.3 for the whole distribution of Democratic candidates in districts where the Democrat won, but for Republican candidates in districts where the Republican won $\hat{j}_x$ is not significantly distinguishable from $\bar{j}$ for $\mathfrak{M}_{12}$ (margin) near zero, rises as $\mathfrak{M}_{12}$ rises and then declines. The latter pattern closely resembles the pattern observed for winners with gerrymandering and turnout decline in the second simulation (Figure 6), but the former resembles the pattern for strategic voting observed in the first simulation. For both sets of losers in Figure 2, the pattern in $\hat{j}_x$ resembles the pattern observed for losers with gerrymandering and turnout decline in the second simulation (Figure 6): for $\mathfrak{M}_{12} \approx 0$, $\hat{j}_x > \bar{j}$, and for high $\mathfrak{M}_{12}$, $\hat{j}_x < \bar{j}$.

The difference between Democratic winners and Republican winners in Figure 2 can be explained by considering Figure 16, which shows second-digit mean results for votes for president in states where the Republican candidate won in the presidential election of 1984 (states where the Democrat won are too few to allow $\hat{j}_x$ to be estimated reliably). In Figure 16, each state is a “district” due to the Electoral College. Values near 4.3 are evident for the Republican candidate. $\hat{j}_x$ is significantly greater than $\bar{j}$ for the Democrat, for $\mathfrak{M}_{12}$ values up to about 0.1. The pattern for the Democrat resembles the pattern for nonstrategic votes in the third simulation (Figure 7), while the pattern for the Republican resembles the pattern from the first simulation that diagnoses strategic voting. If asymmetric strategic voting is diagnosed for both the winning Republican presidential candidate and for Democratic winnners in House races from the same year, then the overall pattern is close to what we should expect if there is strategic party balancing as described by Alesina and Rosenthal (1995, 1996).

*** Figure 16 about here ***

A similar paired pattern may be observed in 1988. Figure 17 shows that in 1988 $\hat{j}_x > \bar{j}$ over most of the distribution for the Republican presidential candidate in states where the

Republican won. In states where the Democrat won, $\hat{j}_x < \bar{j}$ for $\mathfrak{M}_{12} < .06$ and $\hat{j}_x > \bar{j}$ only where $\mathfrak{M}_{12} > .06$, unlike any of the simulations. There is evidence in favor of strategic voting only for one of the two presidential candidates. The pattern for the Democratic presidential candidate in states where he lost again resembles the pattern for nonstrategic votes in the third simulation (Figure 7). In Figure 18, $\hat{j}_x$ for Democratic House winners resembles the pattern for strategic voting observed in the first simulation while $\hat{j}_x$ for Republican winners again resembles the pattern observed for winners with gerrymandering and turnout decline in the second simulation. Again the overall pattern is close to what we should expect when there is strategic party balancing.

*** Figures 17 and 18 about here ***

The strategic party balancing theory of Alesina and Rosenthal (1995) implies there is no strategic vote switching in midterm House elections, and looking at data from 1986 and 1990 that is what we find. Figure 19, which displays results for House elections in 1986, shows no departures of $\hat{j}_x$ from $\bar{j}$ that cannot be explained as a result of gerrymandering and turnout decline: $\hat{j}_x$ for Republicans is not significantly different from $\bar{j}$ for $\mathfrak{M}_{12} = 0$, then rises to be significantly greater than $\bar{j}$ as $\mathfrak{M}_{12}$ increases, then falls back to be not distinct from $\bar{j}$ for high values of $\mathfrak{M}_{12}$; for losers $\hat{j}_x$ is not significantly different from $\bar{j}$ for low values of the $|\mathfrak{M}_{12}|$ but is significantly below $\bar{j}$ at high values; and for Democratic winners $\hat{j}_x$ is not significantly different from $\bar{j}$. Similar patterns are observed for 1990, in Figure 20, except for Republican winners $\hat{j}_x$ is never significantly different from $\bar{j}$.

*** Figures 19 and 20 about here ***

Turnout apparently does generally decline as a function of margin in the elections of the 1980s in the ways necessary for the simulations to be relevant when interpreting $\hat{j}_x$. Figure 21 uses self-report data from the American National Election Studies (ANES) from years

1984–90 to measure whether a person voted in the House election.[37] "Margin" ($\mathfrak{M}_{12}$) is the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using election returns data from Office of the Clerk (2010). Compensating for the lack of geographic coverage in the ANES data—the ANES sample includes responses from only a subset of congressional districts—is the ability to separate voters by self-described partisanship. Separating by partisanship is important because the asymmetric strategic incentives associated with strategic party balancing may imply Democrats and Republicans respond differently to varying electoral margins. Figure 21 shows nonparametric regressions for turnout plotted against $\mathfrak{M}_{12}$ for each level of party identification.[38] In midterm election years, turnout always eventually declines as $\mathfrak{M}_{12}$ moves away from zero, but immediately near $\mathfrak{M}_{12} = 0$ there is a slight increase among Democrats and among Independents as $\mathfrak{M}_{12}$ increases and a slight increase among Republicans as $\mathfrak{M}_{12}$ decreases.[39] Perhaps these slight and asymmetric deviations from a strict pattern of decline reflect the actions of elite political actors (Cox and Munger 1989; Caughey and Sekhon 2011)

*** Figure 21 about here ***

The patterns in $\hat{j}_x$ for the 2008 presidential election, in Figure 1, clearly reflect asymmetric strategic voting in favor of the Democrat. $\hat{j}_x$ persistently having a value of about 4.3 for the Democratic candidate in states where the Democrat won while $\hat{j}_x$ is not significantly different from $\bar{j}$ for the Republican in those same states matches the pattern

[37] A person is counted as having voted in the House election if the response was "yes" to the question, "How about the election for the House of Representatives in Washington. Did you vote for a candidate for the U.S. House of Representatives?" and was not validated as having not voted. Someone who said "yes" but was validated as not voting is coded as not having voted in the House election (Miller and the National Election Studies 1982, 1986, 1989; Miller, Rosenstone and the National Election Studies 1993).

[38] Strong and weak "Democrats" and "Republicans" are counted as respectively Democrats and Republicans, and all kinds of "Independents" are counted as Independents.

[39] The pattern apparent in Figure 21—in which in presidential election years, turnout always declines as $\mathfrak{M}_{12}$ falls below zero, but it increases slightly and then declines as $\mathfrak{M}_{12}$ increases above zero—does not match the pattern in aggregate data for all districts. When turnout is defined as the ratio of the sum of votes cast for either the Democrat or Republican candidate in each race divided by the voting age population, turnout in 1984 and in 1988 increases slightly as $\mathfrak{M}_{12}$ decreases below zero, before it declines. As $\mathfrak{M}_{12}$ increases above zero, turnout declines. In 2008 turnout is flat for $-.25 \leq \mathfrak{M}_{12} \leq .3$, then it declines as $|\mathfrak{M}_{12}|$ increases further.

from the first simulation that diagnoses strategic voting for one candidate but not the other. Similar values of $\hat{j}_x$ are not observed for the Republican candidate in states where the Republican won, while $\hat{j}_x$ is not significantly different from $\bar{j}$ for the Democrat in those states.

While Figure 1 reflects asymmetric strategic voting in favor of the Democrat, Figure 22, which shows results for House elections that year, suggests it is Democratic House winners and not Republican winners who seem to benefit from asymmetric strategic voting. In this case $\hat{j}_x$ is not significantly greater than $\bar{j}$ for all Democratic candidates. House losers of both parties have $\hat{j}_x$ values matching those for losers who received no strategic votes. This asymmetric pattern, which suggests strategic voting for Democrats both for House and President, does not match the predictions of Alesina and Rosenthal's theory with homogeneous parties but may be predicted by a version with both "mainstream" and more extreme Tea Party Republican factions.

*** Figure 22 about here ***

Heterogeneous parties may also explain the patterns apparent in Figure 23 for House elections in 2006. The patterns in digits in 2006 suggest asymmetric strategic voting that favors Democrats. The estimates of $\hat{j}_x$ for Republican winners and losers and for Democratic losers all resemble the patterns produced by gerrymanders in the simulations. By 2008 the tension between the Tea Party and "mainstream" Republicans was clear. But perhaps such divisions were already affecting many votes in 2006.

*** Figure 23 about here ***

The evidence for the strategic party balancing theory does not imply that strategic voting according to wasted-vote logic does not occur. There is evidence in favor of strategic voting in presidential elections from a test of the "bimodality" hypothesis introduced in Cox (1994): if there is a Duvergerian equilibrium so that the $M + 1$ rule holds, then the ratio of the second loser's vote total to the first loser's vote total should be approximately zero. In presidential elections in the 1980s, 2004 and 2008 this relationship holds in all states, and in 2000 it holds in all states except Alaska. In 1992 and other years with a prominent third-party presidential candidate, of course, bimodality test results do not support the predictions of the $M+1$ rule. But the prevalence of $\hat{j}_x$ values that are interpretable in terms of asymmetric strategic voting and gerrymander (with rolloff) shows that any effects vote switching motivated by wasted-vote logic may have on the vote counts' digits are dominated by the patterns induced by partisan districting and by the large-scale strategic coordination described by Alesina and Rosenthal (1995).

With one big difference, patterns for state legislative election data resemble those observed for U.S. House elections. A quick look at the data suggests that during the 1980s this does not happen with the kinds of strategic adjustments that would tie state legislatures to the president as happens with the federal legislature and the president in the theory of Alesina and Rosenthal (1995). Figure 24, which shows $\hat{j}_x$ for state house and state senate data pooled over years 1984, 1986, 1988 and 1990, strongly resembles the pattern for what happens under gerrymandering with turnout decline and no strategic voting. Fiorina's theory does not depend on strategic behavior, but the entanglement across levels of government that would support a strategic version of Fiorina's argument about federalism may be true during the late 2000s. Figure 25, which shows $\hat{j}_x$ for 2006 and 2008 state house and state senate elections, suggests asymmetric strategic voting in favor of the Democrats.

*** Figures 24 and 25 about here ***

Discussion

The second significant digits in precinct-level vote counts typically have distinctive values when voters are switching their votes as a result of strategic calculations. The digits also have different distinctive patterns when a gerrymander is affecting the vote counts. The digits have yet another distinctive pattern—the one described by 2BL—when none of these things are happening. The pattern when there is coercion is distinctive yet again. Merely by using the patterns in the digits in vote counts, without having any information about preferences or about beliefs, it is possible to diagnose when strategic voting and otherwise normal features of electoral politics are occurring. Against this background, election fraud in the form of coercion can stand out. The 2BL pattern alone is not enough, but knowledge of the more elaborate patterns that occur under normal conditions can support an election forensics exercise.

Not only vote switching due to wasted-vote logic considerations but much more elaborate kinds of strategy—like the strategies induced by the equilibria described by Alesina and Rosenthal (1996)—can be detected using vote counts' second digits. The diagnostic power of the digits is not peculiar to one kind of electoral system, but knowledge of the electoral system can inform the kinds of covariates needed to understand the digit data. The margin between candidates or parties in the election is always useful, but often there are several possible margins, and knowledge of the electoral system can guide the choice of which one is most appropriate. Party labels matter, and knowledge of what governing coalitions may form or of other aspects of how the government will work—separation of powers?—can motivate the definition of covariates like $\mathfrak{D}_k$ and inform appropriate decisions about how to subset the vote data. The more one knows, the better the analysis will be. But data regarding voters' preferences or beliefs is not necessary.

Digits can help diagnose voters' strategies, but whether they can reliably help detect election fraud remains an open question. One consideration is that data from many precincts in several districts are needed to identify the pattern in the digits, and to obtain statistically reliable results the sample size of districts may need to be considerable. While it is always possible to speculate about the meaning of the digit means from one or a few districts, if based only on what is known about these kinds of electorally significant digit distributions as of now, such speculation must remain just that—conjectural.

The technology discussed in this paper can reliably help detect only some kinds of widespread corruption. The election in Iran in 2009 is an example. In “ballot box” (polling station) data Mebane (2010*b*) found digit-based evidence suggesting that two candidates with very low vote totals had been strategically abandoned, while the winning candidate had a distribution of votes and vote-count digits that strongly suggested there had been extensive ballot box stuffing in the candidate’s favor. For the latter diagnosis, the availability of counts of the number of invalid ballots in each ballot box was essential. On the other hand, from Mebane (2006) and unreported simulations similar to those in the current paper, fakery in the form of a strictly proportional increase or decrease in the votes for a candidate would generally not be detected by digit tests. It is likely there are many other forms of election fraud that would fail to be detected by digit tests. Fraud affecting only a few isolated precincts could not be detected.

The simulation in this paper is based on a mixture process that generates individual preferences that, when aggregated into precincts, approximately satisfy 2BL. By deriving nonstrategic and then strategic and gerrymandered and then coerced votes from these preferences, I find that tests based on the second significant digits of the precinct counts are sensitive to differences in how the counts are derived. Tests using the second-digit mean are more useful for diagnosis than tests using a chi-squared statistic that is based directly on Benford’s Law. The tests can sometimes distinguish the effects of coercion—where votes are cast regardless of preferences—from the effects of strategic voting and gerrymanders, and strategic from nonstrategic voting.

These findings based on simulations support plausible interpretations of real data. The simulated digit patterns match the patterns in German Bundestag elections that are expected according to theories and previous empirical findings regarding the kinds of strategies voters are using in those elections. The upper bound on the second-digit means, $\hat{j}_{xy}$, observed in the real election data exceed the upper bound in the simulation, but this seems due to the real data having local party imbalances—gerrymanders—more extreme than were considered in the simulation. The digit tests confirm previous findings that strategic party balancing occurred in the American national elections of the 1980s. In 2006 and 2008, however, more complicated voter behavior is apparent, due possibly to the increasing divisions that developed in the Republican party. In the latter two years, strategic voting seems to have benefited the Democrats in both the presidential and legislative elections. During these years the digit tests also suggest a strategic pattern developed that connects national election outcomes to election outcomes in state elections.

Using digit tests to understand the consequences of strategic voting and gerrymandering and to diagnose possible fraud depends on the availability of suitable covariates. In the American case the margin in each jurisdiction and the party of the apparent winner are the covariates used to estimate the conditional second-digit mean $\hat{j}_x$. In Germany data about margins, party labels and the differences between *Erststimmen* and *Zweitstimmen* votes are used to estimate $\hat{j}_{xy}$. In elections where the simulations are informative, the explanation they provide for the second-digit patterns is qualitative: not every variation in $\hat{j}$ is accounted for. The strategic voting and gerrymander mechanisms considered here matter, but evidently they are not all that's going on.

Except in the vaguest way, by citing work such as Rodriguez (2004) and Grendar, Judge and Schechter (2007), this paper does not explain why precinct-level vote counts so often satisfy 2BL, which, empirically, they very often do. Nonetheless, given that starting point, the evidence is strong that departures from 2BL, which also occur frequently, are related both to normal political phenomena and to serious election anomalies.

Much remains to do. One desirable task is to replace the simulations with deductive, analytical arguments. Here I briefly offer some musings about this.[40] Deductive arguments will be difficult to construct. Berger (2005) proves that numbers that represent multidimensional dynamical systems usually satisfy Benford's Law. The exception is systems that are degenerate in a special way, notably those that have zero eigenvalues in

[40] In fact something like these ideas motivated the data gathering and then the simulation discussed above.

their linearizations (so-called "exponentially b-resonant spectra," Berger (2005, 228)). In an argument that focuses on the topology of mappings, Schofield (1983) demonstrates that the only sets of preferences for which majority rule cycles do not exist are those with severely limited dimensionality (2 or 3). Preference dimensions beyond those necessarily allow a "dense" set of cycles to exist (Schofield 1983, 702). It is in such circumstances with high-dimensional preferences, which arguably always hold in practice, that strategic voting can affect outcomes. Majority rule thus in some sense represents a degenerate mapping. Deductive theory to demonstrate when certain voting rules, preferences and election strategies imply particular values for the second digits of vote counts will probably need to draw together theories of these types—a combination of number theory, social choice theory and statistics.

Digits alone are about as minimal a foundation for drawing inferences about what happened in elections as might be imagined. If all one has are vote counts and consequently their digits, then there is no information about preferences, strategies, campaigns or anything else that one would normally use to try to understand what went on in an election. Given appropriate covariates, tests based on vote counts' digits can do a lot to give strong suggestions about what happened.

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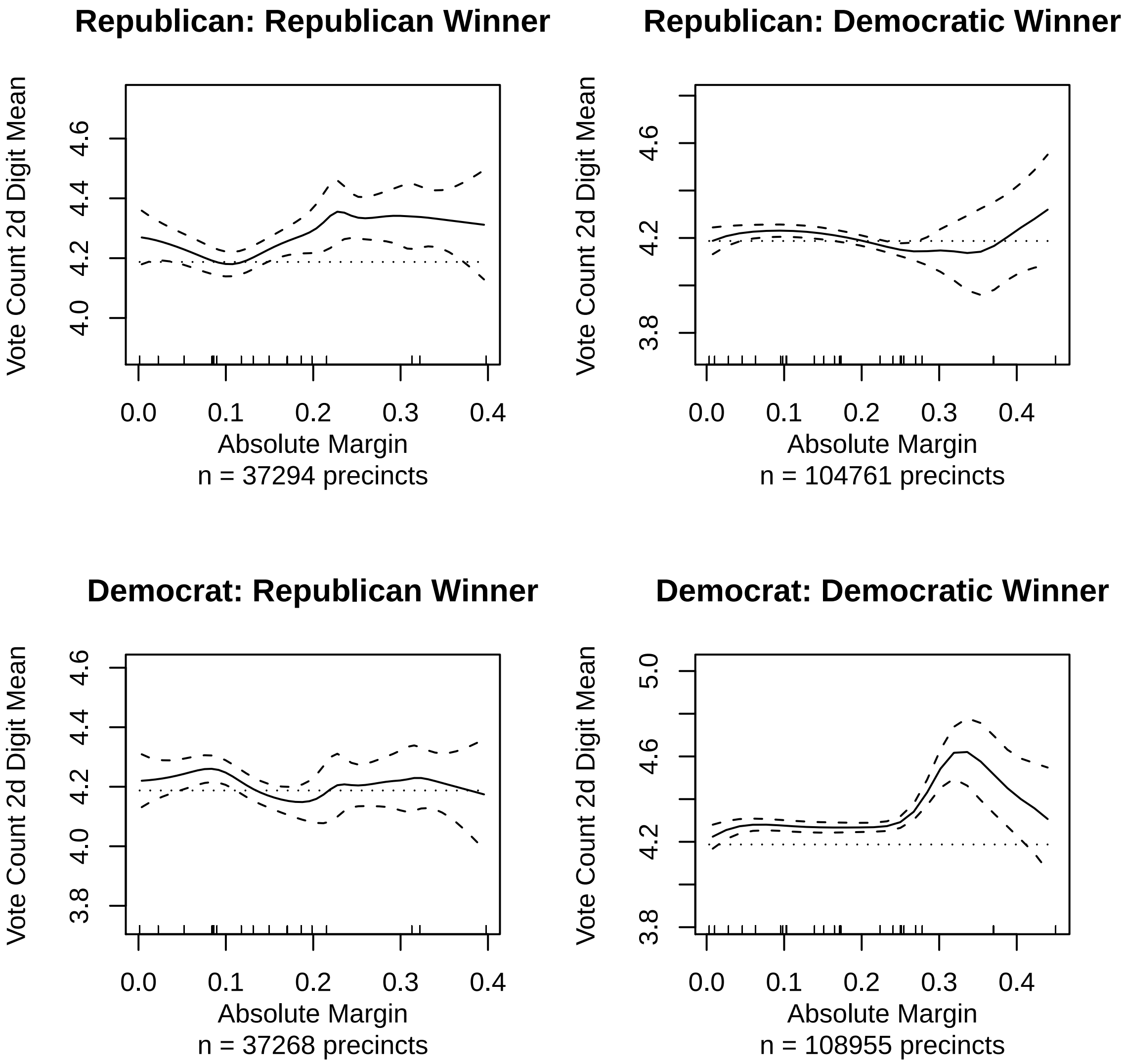
Table 1: Second-digit χ^2_{2BL} statistics, means, standard errors and "vote" totals: asymmetric four-candidate simulation

	y_1	y_2	y_3	y_4	w_1	w_2	w_3	$\tilde{w}_1$
χ^2_{2BL}	10.7	12.6	11.9	12.6	12.3	12.2	951.1	58.0
$\hat{j}$	4.29	4.15	4.32	4.32	4.35	4.35	2.68	3.75
s.e.	.040	.041	.041	.040	.041	.041	.043	.042
votes	$200,284$	$271,628$	$181,172$	$163,970$	$329,043$	$310,300$	$13,741$	$493,013$

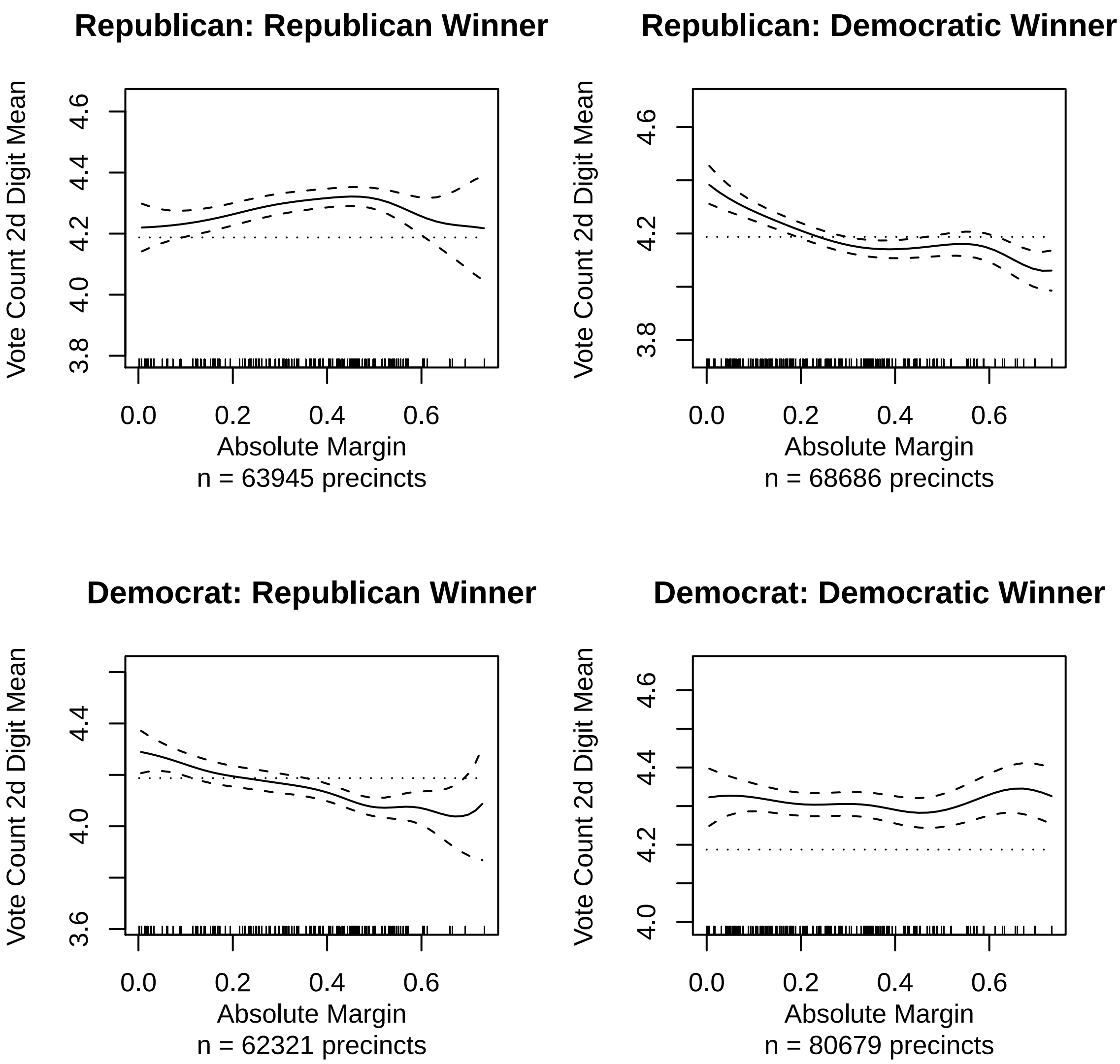
Note: $n = 5000$ precincts. $\mathcal{N} = 1300$, $\sigma = 1$, $\upsilon = 1.75$, $t = 0.15$, 500 replications.

Figure 1: Vote Counts for President, 2008



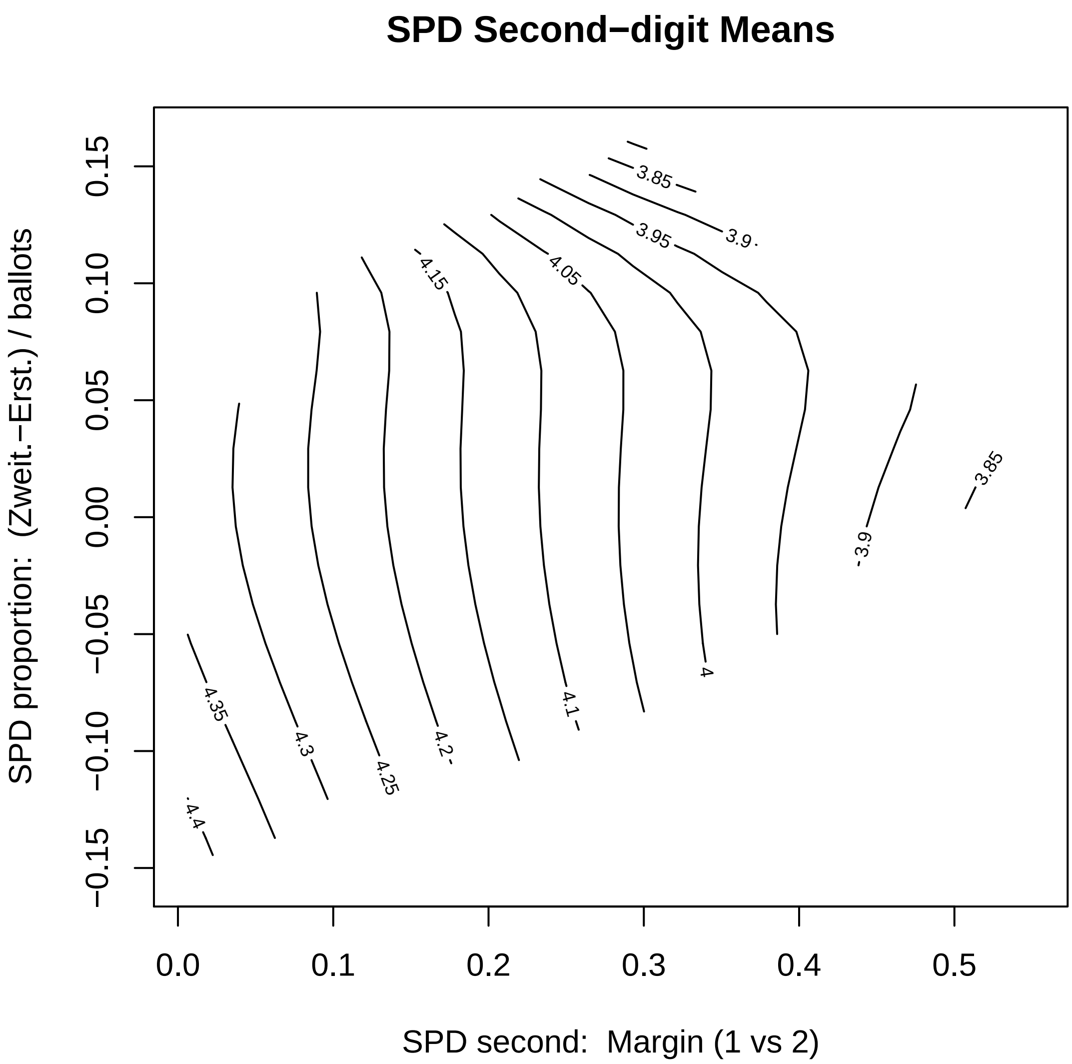
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the total of votes cast for president, using U.S. House Clerk official election returns data. Rug plots show the locations of state absolute margins.

Figure 2: Vote Counts for United States Representative, 1984



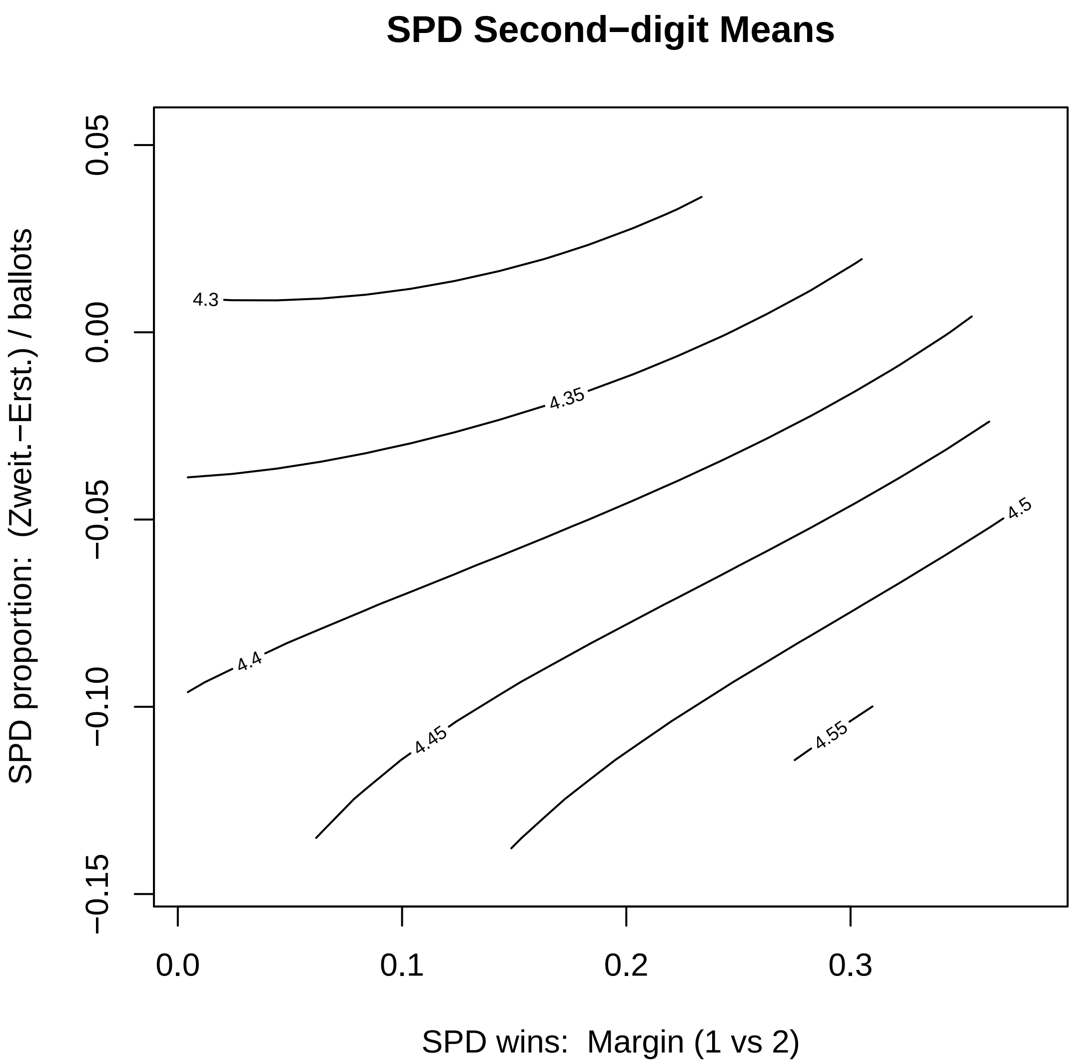
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on ROAD precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district absolute margins.

Figure 3: Polling Station Vote Counts for SPD, 2002–2009



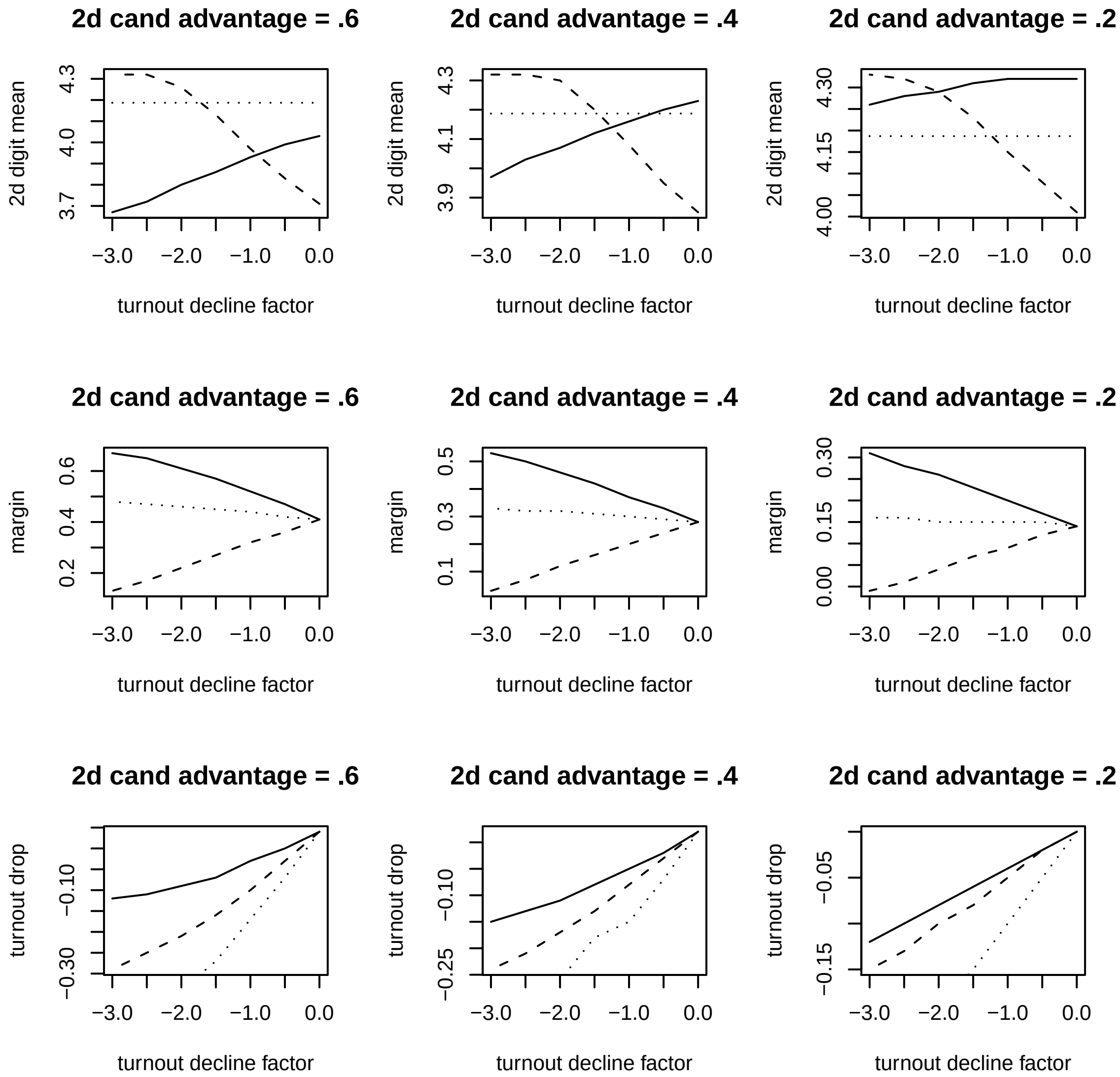
Note: Nonparametric regression contours based on polling station data, using polling stations in *Wahlkreise* where SPD had the second most *Erststimmen* votes. "Margin" is the number of *Erststimmen* votes for SPD in each *Wahlkreis* subtracted from the number of votes for the first-place party divided by the total of *Erststimmen* votes cast in the *Wahlkreis*. The "SPD proportion" is the total of *Zweitstimmen* votes cast for SPD minus the number of *Erststimmen* votes cast for SPD divided by the total number of ballots used in the *Wahlkreis*.

Figure 4: Polling Station Vote Counts for SPD, 2002–2009



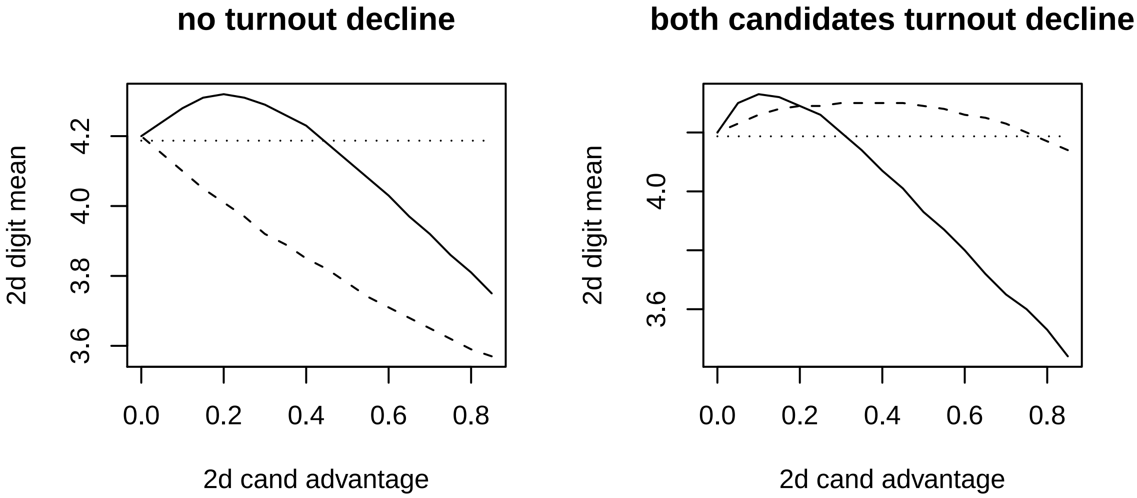
Note: Nonparametric regression contours based on polling station data, using polling stations in *Wahlkreise* where SPD had the most *Erststimmen* votes. "Margin" is the number of *Erststimmen* votes for SPD in each *Wahlkreis* minus the number of votes for the second-place party divided by the total of *Erststimmen* votes cast in the *Wahlkreis*. The "SPD proportion" is the total of *Zweitstimmen* votes cast for SPD minus the number of *Erststimmen* votes cast for SPD divided by the total number of ballots used in the *Wahlkreis*.

Figure 5: Second-digit means, margins and turnout drop proportions by turnout decline: two-candidate simulation



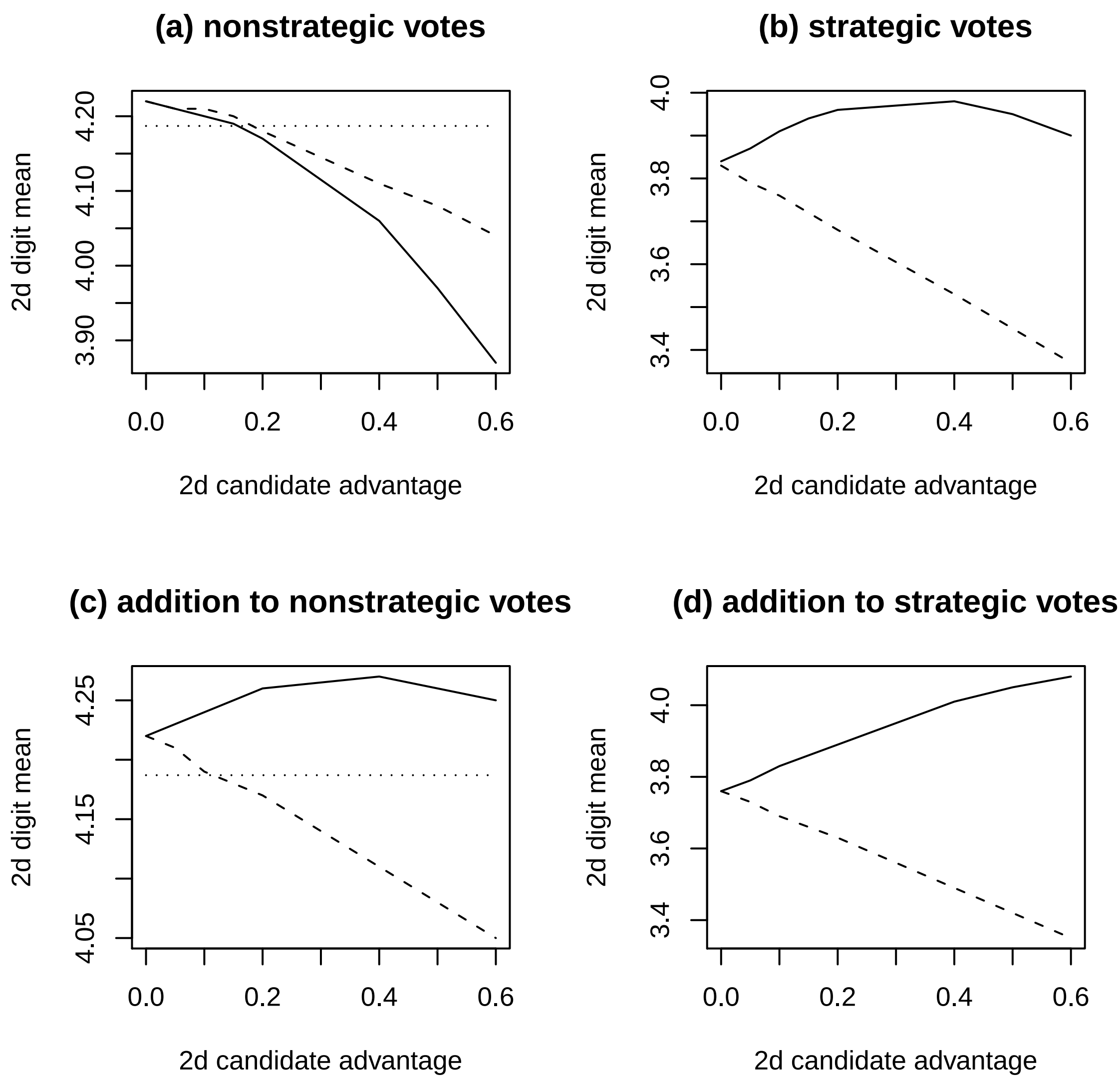
Note: In second-digit mean plots, solid line is first candidate (disadvantaged), dashed line is second candidate (advantaged) and dotted line is mean expected under Benford's Law. In margin and turnout drop plots, solid line is margin or turnout drop as only first candidate's turnout factor increases, dashed line is margin or turnout drop as only second candidate's turnout factor increases and dotted line is margin or turnout drop as both candidates' turnout factor increases.

Figure 6: Second-digit means by candidate advantage: two-candidate simulation



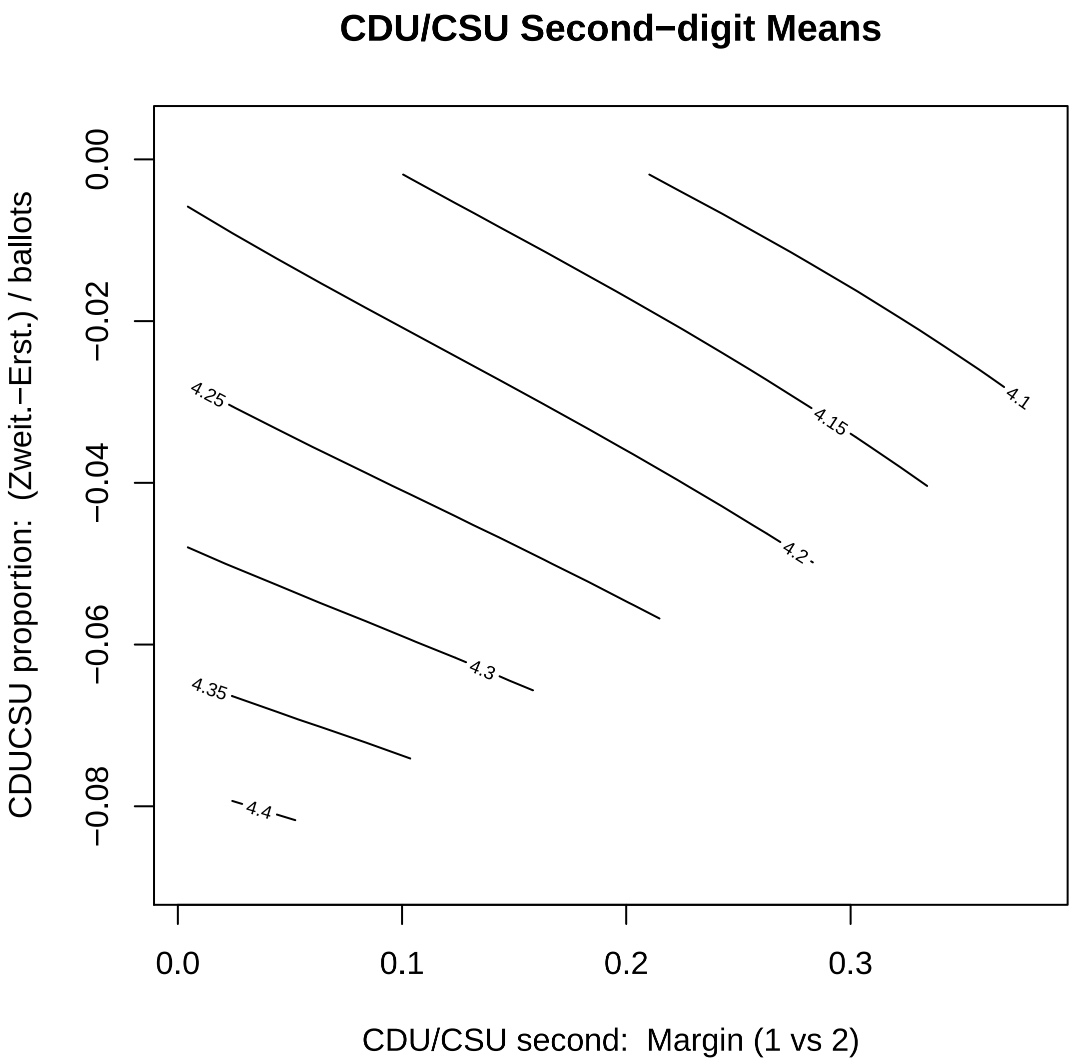
Note: In rightmost graph, turnout decline factor $= -2$. Solid line is first candidate (disadvantaged). Dashed line is second candidate (advantaged). Dotted line is mean expected under Benford's Law.

Figure 7: Second-digit means by candidate advantage (0 turnout decline factor): symmetric four-candidate simulation including coercion



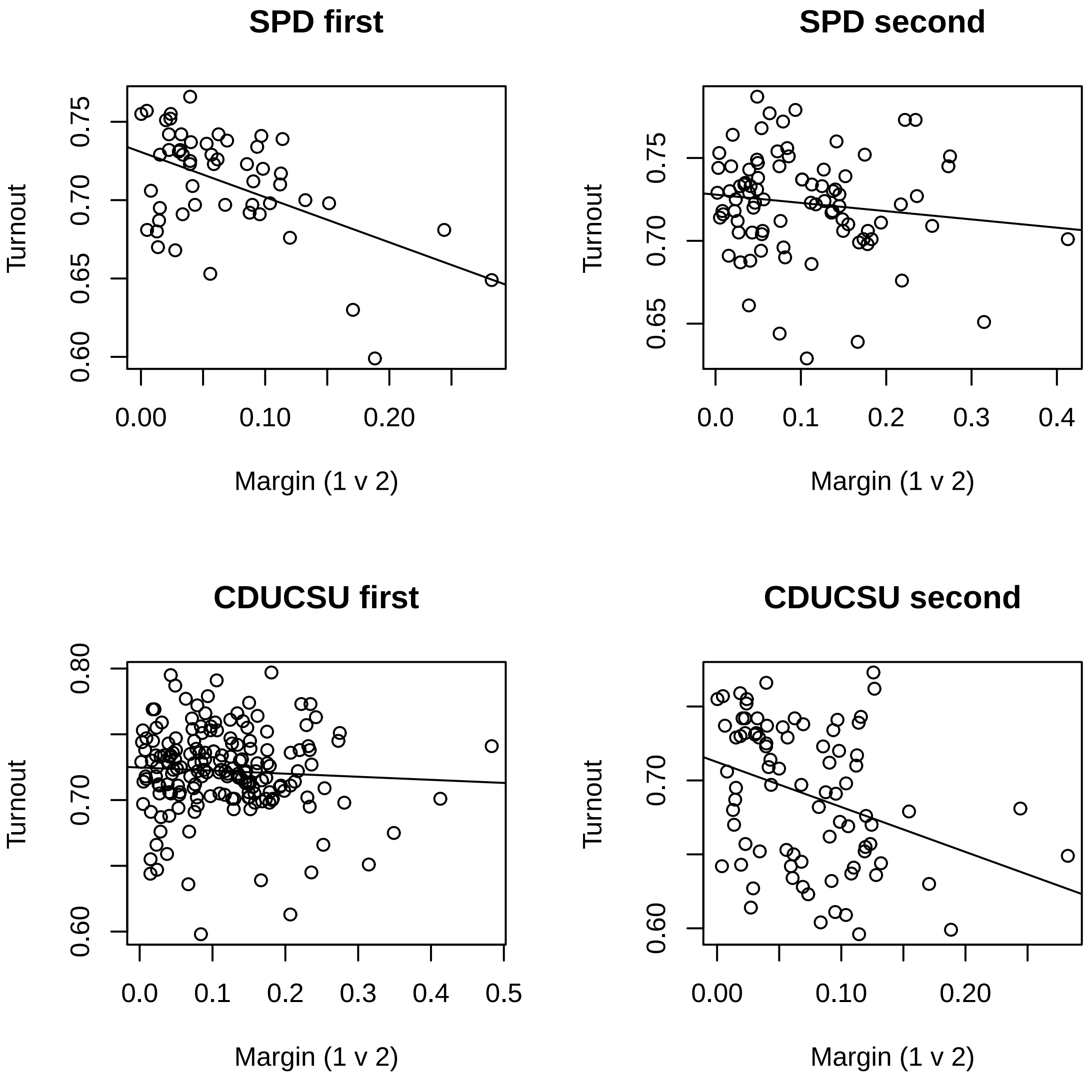
Note: Solid line is first candidate (disadvantaged). Dashed line is second candidate (advantaged). Dotted line is mean expected under Benford's Law.

Figure 8: Polling Station Vote Counts for CDU/CSU, 2002–2009



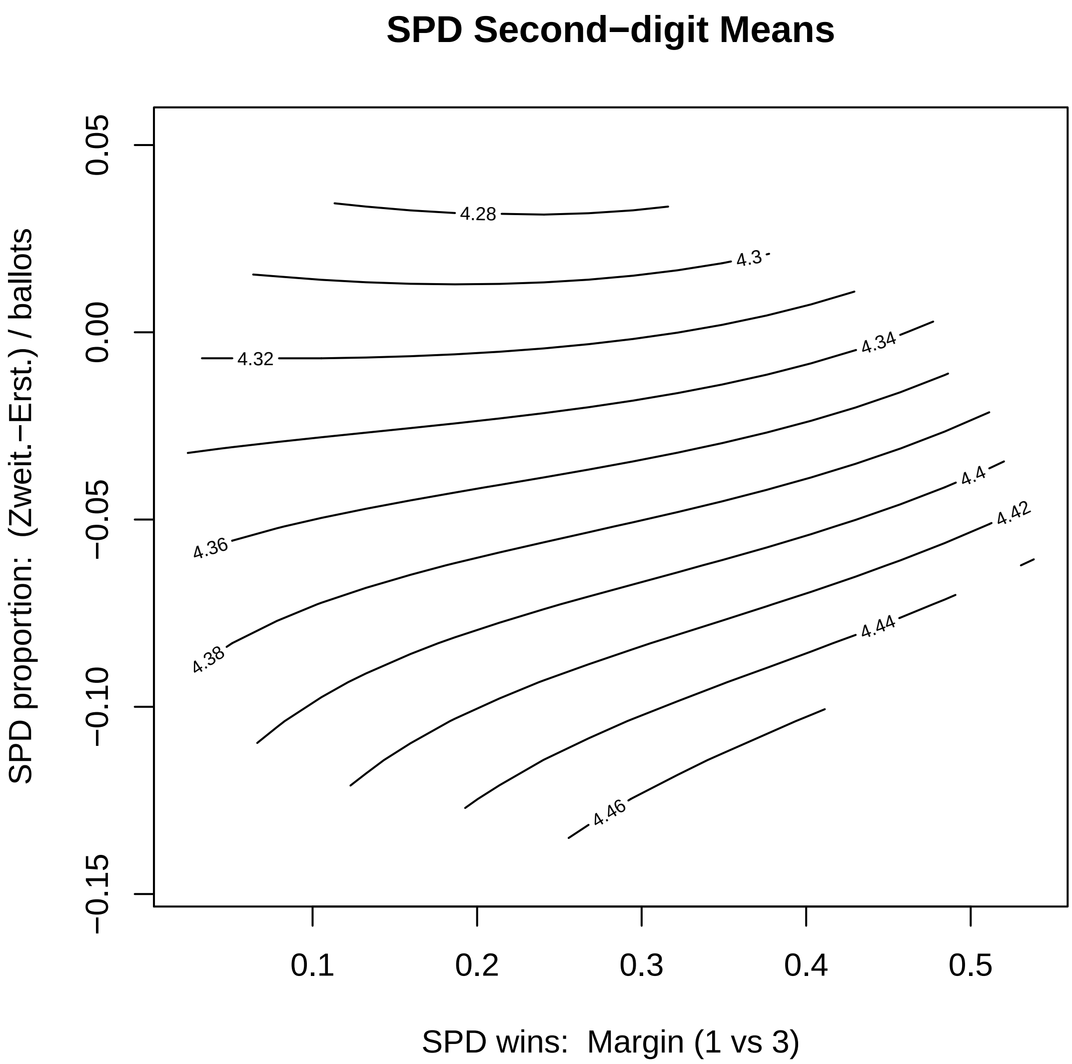
Note: Nonparametric regression contours based on polling station data, using polling stations in *Wahlkreise* where CDU/CSU had the second most *Erststimmen* votes. "Margin" is the number of *Erststimmen* votes for CDU/CSU in each *Wahlkreis* subtracted from the number of votes for the first-place party divided by the total of *Erststimmen* votes cast in the *Wahlkreis*. The "CDU/CSU proportion" is the total of *Zweitstimmen* votes cast for CDU/CSU minus the number of *Erststimmen* votes cast for CDU/CSU divided by the total number of ballots used in the *Wahlkreis*.

Figure 9: Turnout by Vote Margin, Germany, 2009



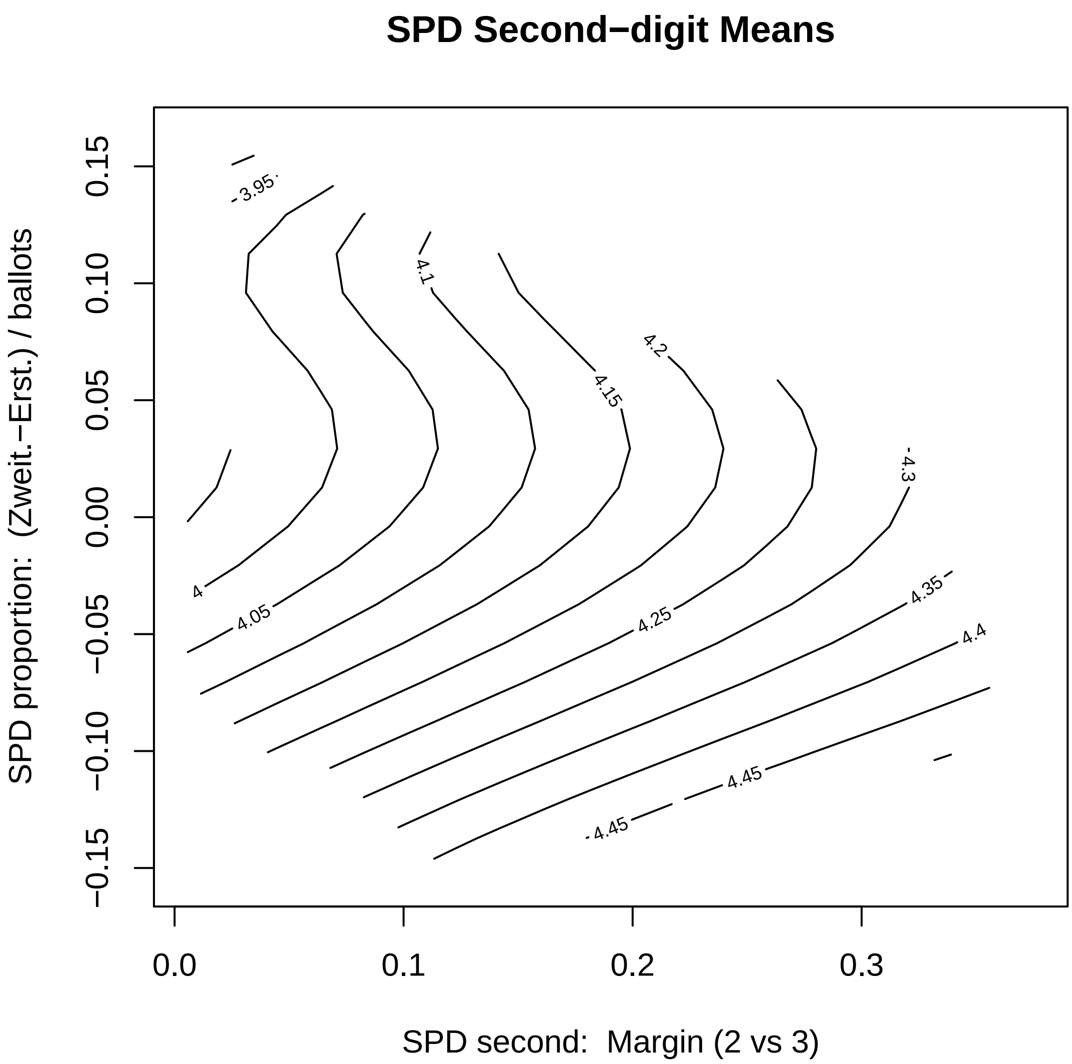
Note: Scatterplot and regression line for *Wahlkreis* turnout by margin between first-place and second-place parties in *Erststimmen* votes. "SPD first" indicates *Wahlkreise* where SPD finished first in the *Erststimmen* votes, "SPD second" indicates where SPD finished second, etc.

Figure 10: Polling Station Vote Counts for SPD, 2002–2009



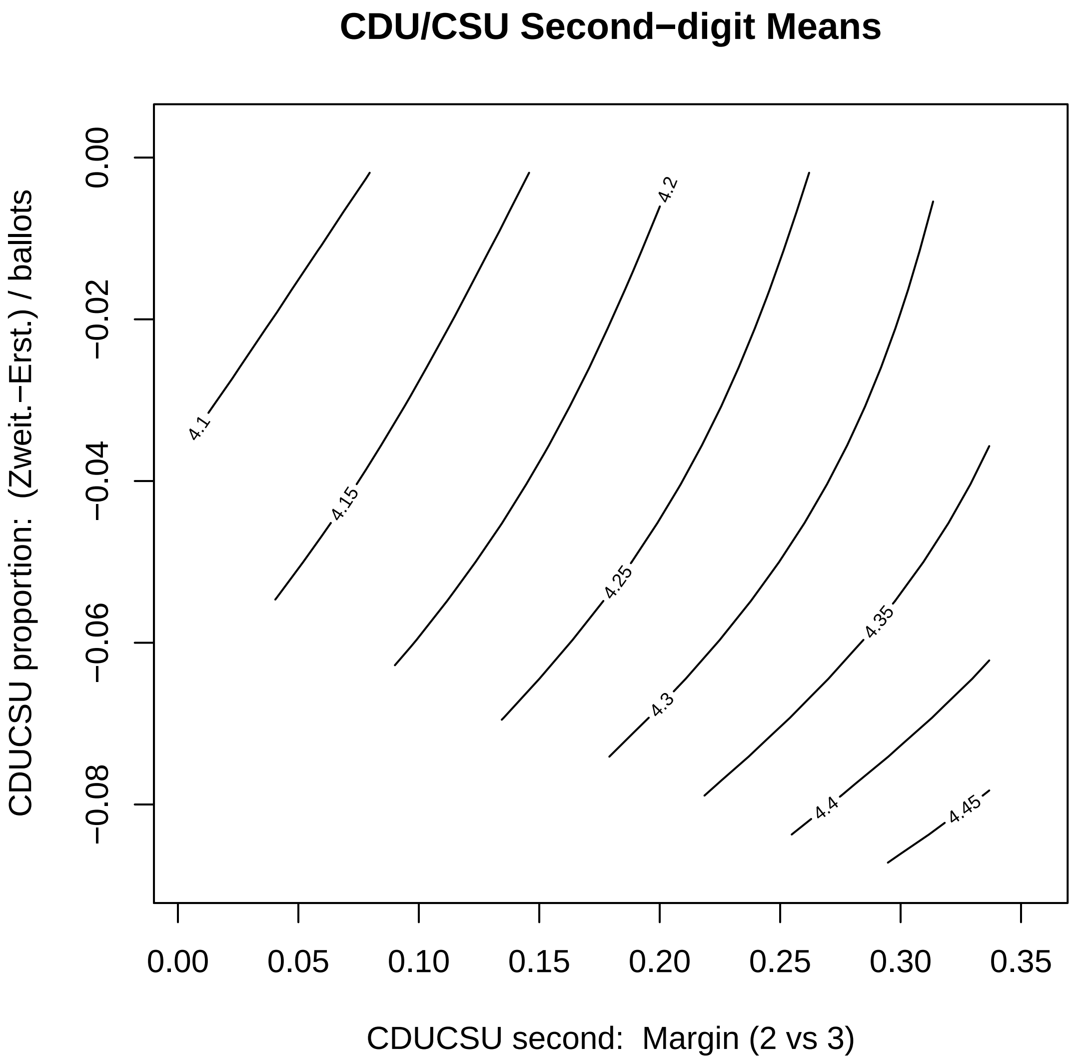
Note: Nonparametric regression contours based on polling station data, using polling stations in *Wahlkreise* where SPD had the most *Erststimmen* votes. “Margin” is the number of *Erststimmen* votes for SPD in each *Wahlkreis* minus the number of votes for the third-place party divided by the total of *Erststimmen* votes cast in the *Wahlkreis*. The “SPD proportion” is the total of *Zweitstimmen* votes cast for SPD minus the number of *Erststimmen* votes cast for SPD divided by the total number of ballots used in the *Wahlkreis*.

Figure 11: Polling Station Vote Counts for SPD, 2002–2009



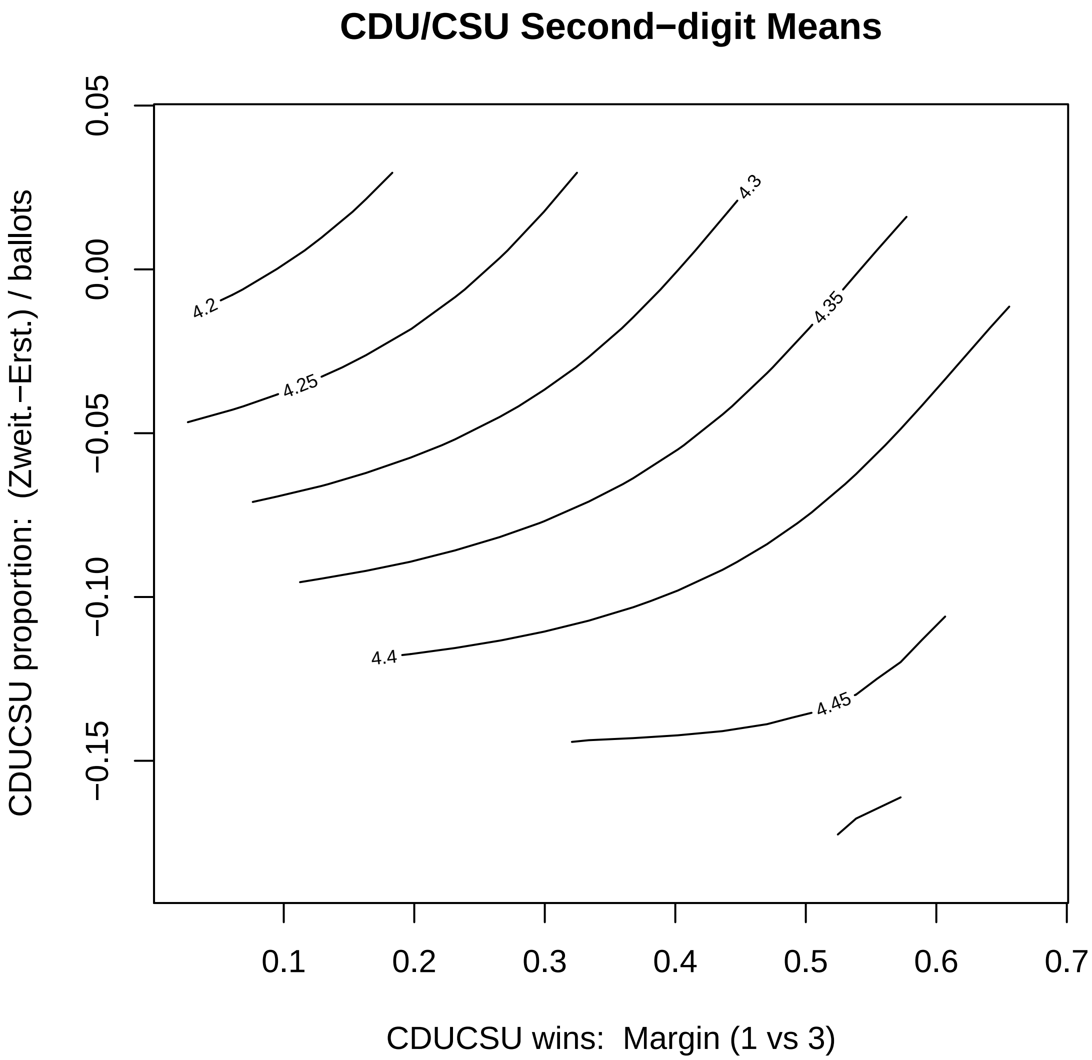
Note: Nonparametric regression contours based on polling station data, using polling stations in *Wahlkreise* where SPD had the second most *Erststimmen* votes. "Margin" is the number of *Erststimmen* votes for SPD in each *Wahlkreis* minus the number of votes for the third-place party divided by the total of *Erststimmen* votes cast in the *Wahlkreis*. The "SPD proportion" is the total of *Zweitstimmen* votes cast for SPD minus the number of *Erststimmen* votes cast for SPD divided by the total number of ballots used in the *Wahlkreis*.

Figure 12: Polling Station Vote Counts for CDU/CSU, 2002–2009



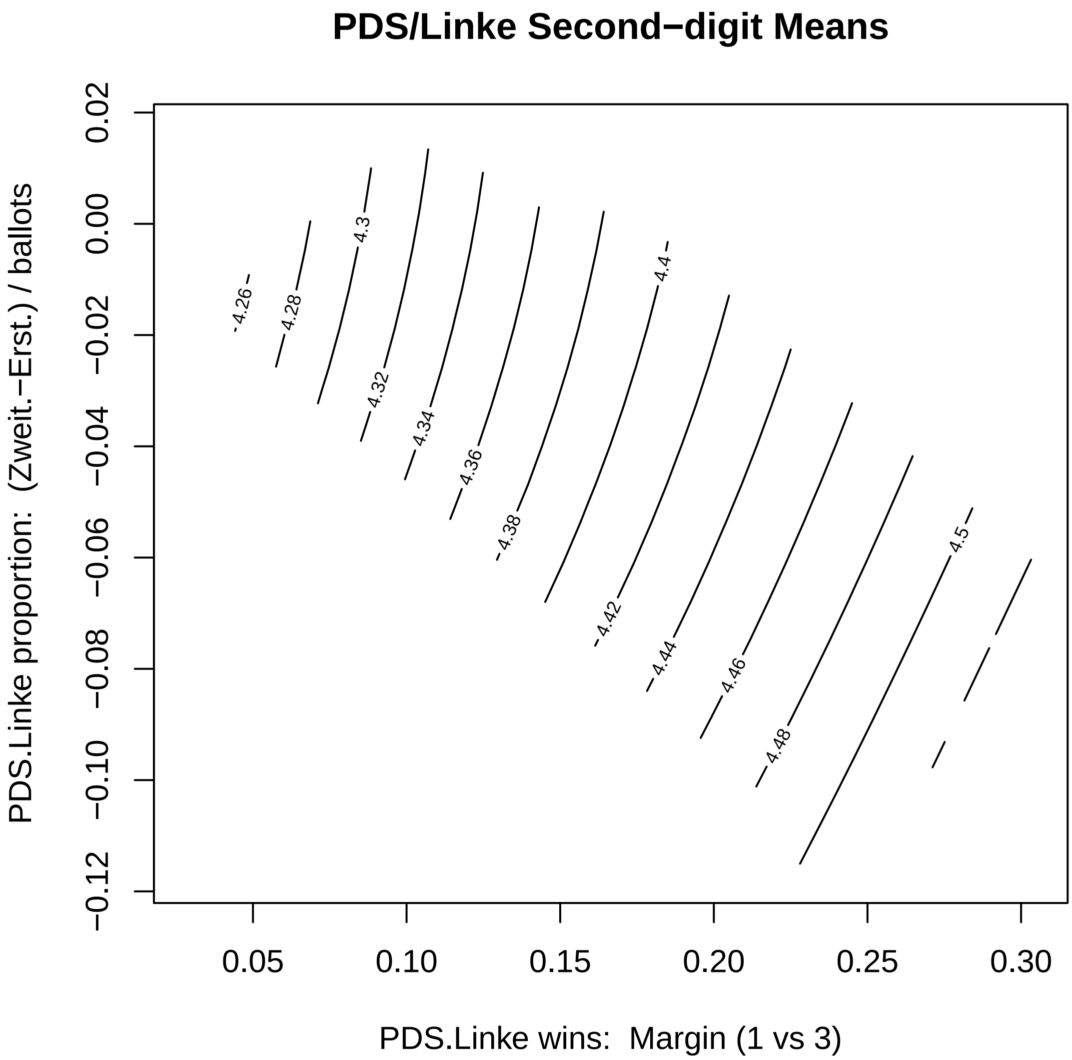
Note: Nonparametric regression contours based on polling station data, using polling stations in *Wahlkreise* where CDU/CSU had the second most *Erststimmen* votes. "Margin" is the number of *Erststimmen* votes for CDU/CSU in each *Wahlkreis* minus the number of votes for the third-place party divided by the total of *Erststimmen* votes cast in the *Wahlkreis*. The "CDU/CSU proportion" is the total of *Zweitstimmen* votes cast for CDU/CSU minus the number of *Erststimmen* votes cast for CDU/CSU divided by the total number of ballots used in the *Wahlkreis*.

Figure 13: Polling Station Vote Counts for CDU/CSU, 2002–2009



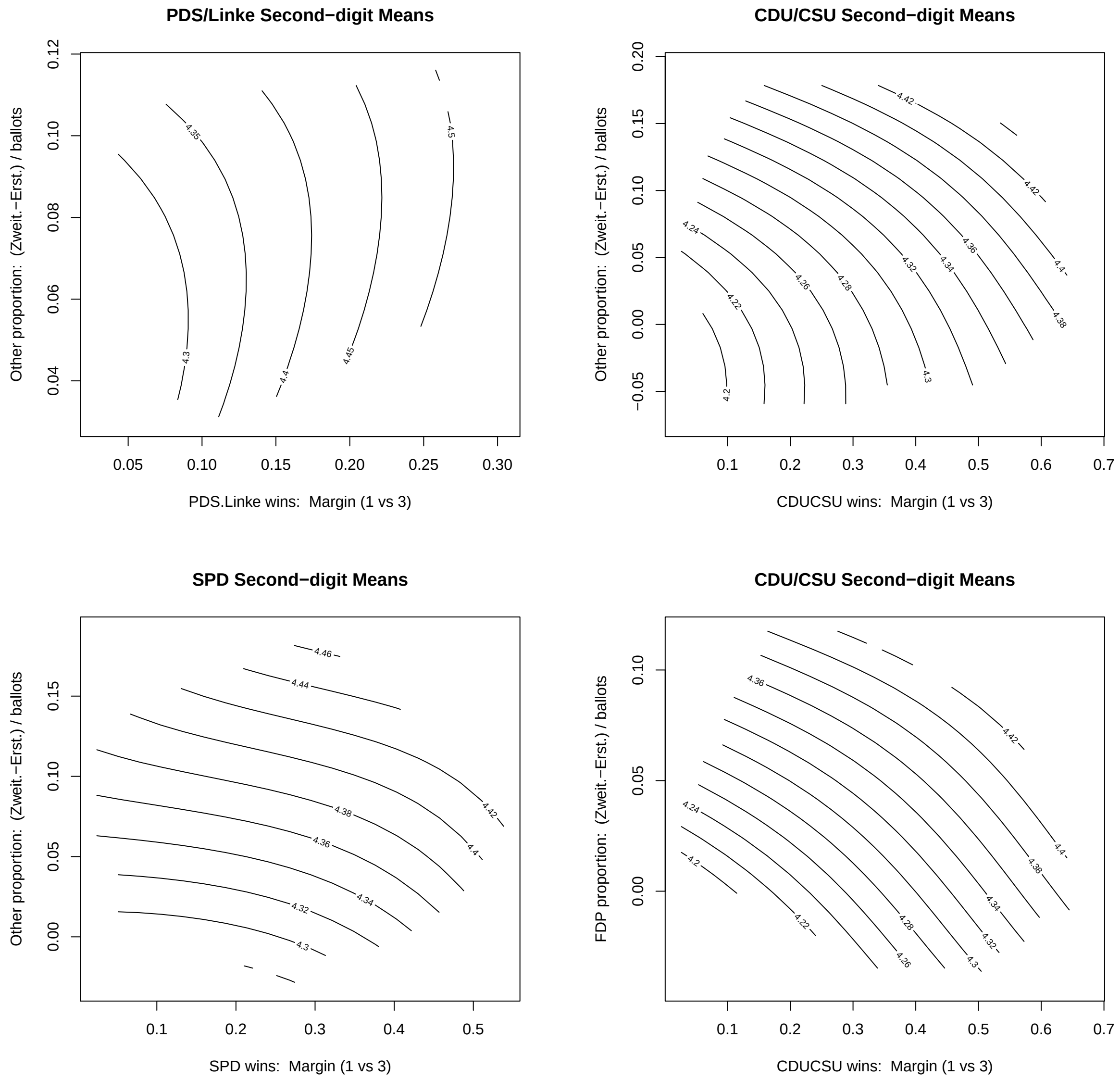
Note: Nonparametric regression contours based on polling station data, using polling stations in *Wahlkreise* where CDU/CSU had the most *Erststimmen* votes. "Margin" is the number of *Erststimmen* votes for CDU/CSU in each *Wahlkreis* minus the number of votes for the third-place party divided by the total of *Erststimmen* votes cast in the *Wahlkreis*. The "CDU/CSU proportion" is the total of *Zweitstimmen* votes cast for CDU/CSU minus the number of *Erststimmen* votes cast for CDU/CSU divided by the total number of ballots used in the *Wahlkreis*.

Figure 14: Polling Station Vote Counts for PDS/Linke, 2002–2009



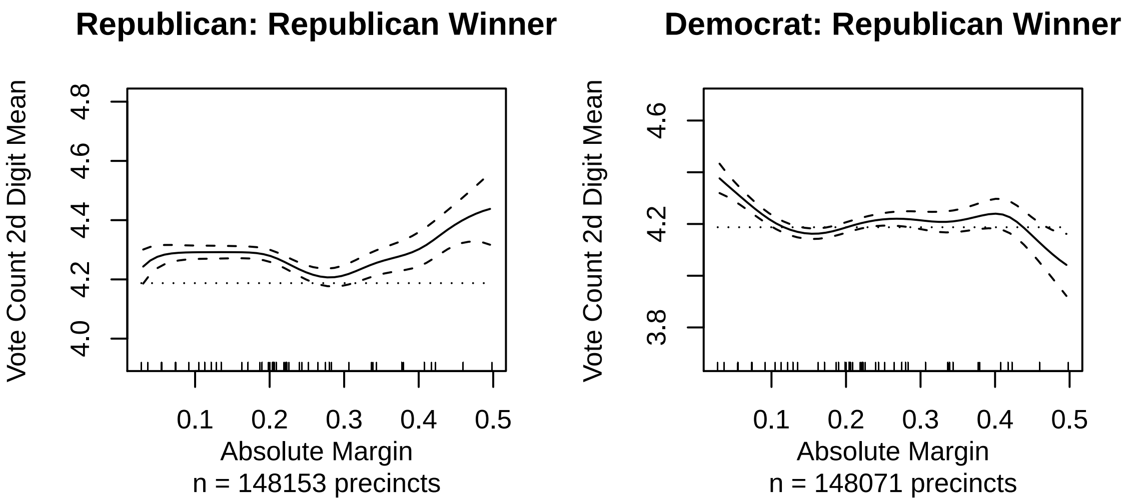
Note: Nonparametric regression contours based on polling station data, using polling stations in *Wahlkreise* where PDS/Linke had the most *Erststimmen* votes. "Margin" is the number of *Erststimmen* votes for PDS/Linke in each *Wahlkreis* minus the number of votes for the third-place party divided by the total of *Erststimmen* votes cast in the *Wahlkreis*. The "PDS/Linke proportion" is the total of *Zweitstimmen* votes cast for PDS/Linke minus the number of *Erststimmen* votes cast for PDS/Linke divided by the total number of ballots used in the *Wahlkreis*.

Figure 15: Polling Station Vote Counts for PDS/Linke, 2002–2009



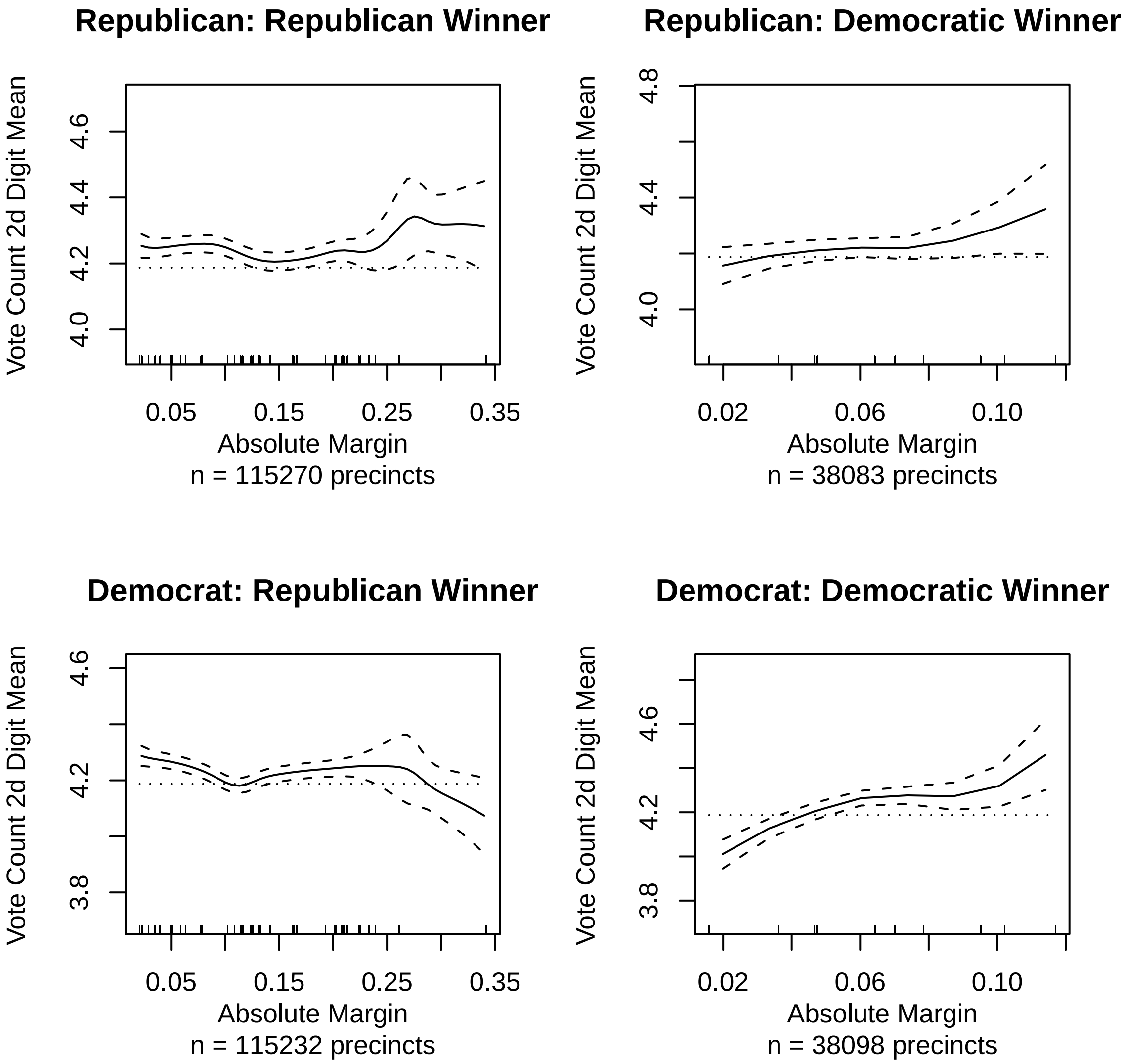
Note: Nonparametric regression contours based on polling station data, using polling stations in *Wahlkreise* where the party names as the party that "wins" had the most *Erststimmen* votes. "Margin" is the number of *Erststimmen* votes for the first-place party in each *Wahlkreis* minus the number of votes for the third-place party divided by the total of *Erststimmen* votes cast in the *Wahlkreis*. The "Other proportion" is the total of *Zweitstimmen* votes cast for parties other than SPD, CDU/CSU or PDS/Linke minus the number of *Erststimmen* votes cast for such parties divided by the total number of ballots used in the *Wahlkreis*. The "FDP proportion" is the total of *Zweitstimmen* votes cast for FDP minus the number of *Erststimmen* votes cast for FDP divided by the total number of ballots used in the *Wahlkreis*.

Figure 16: Vote Counts for President, 1984



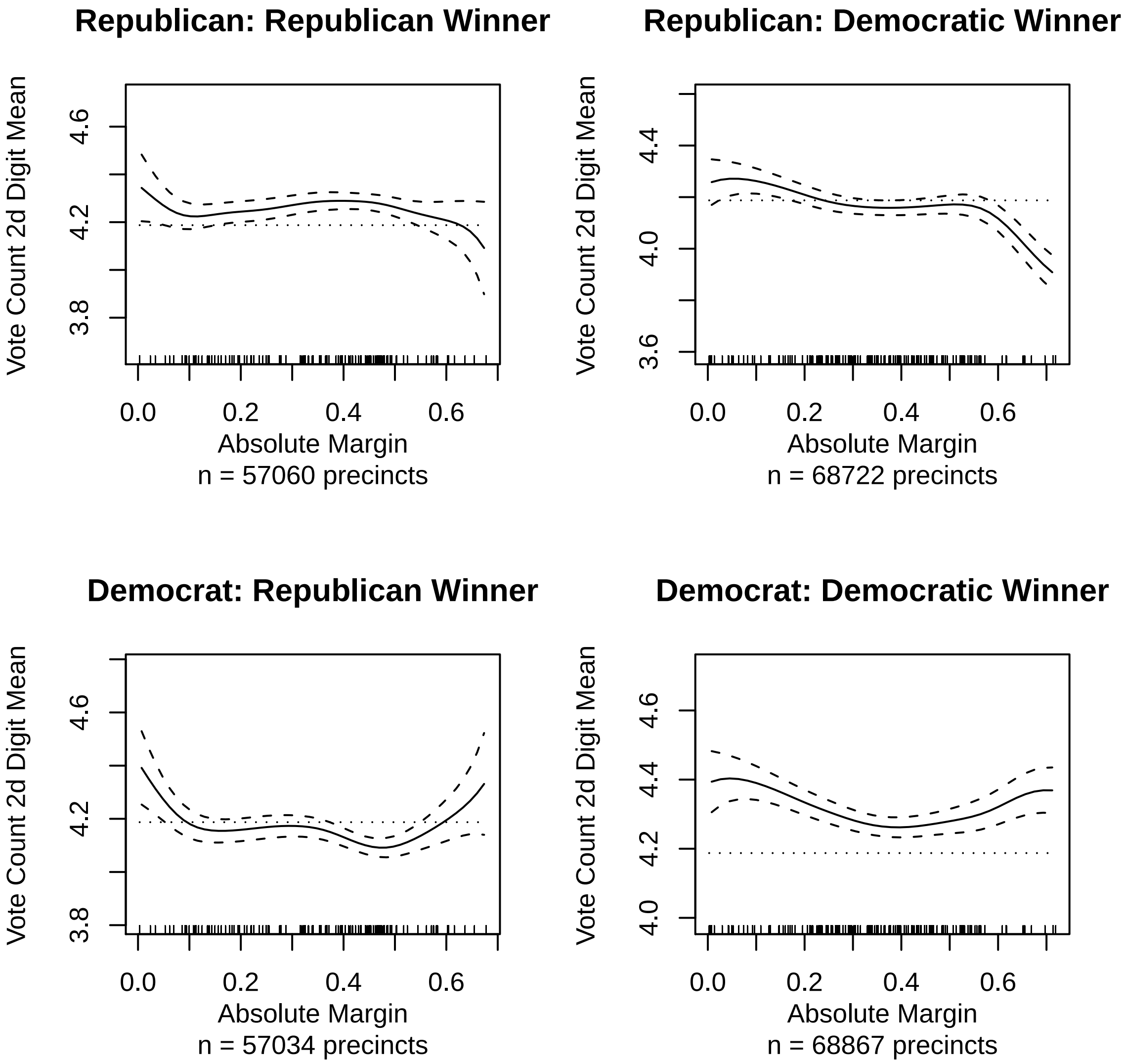
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on ROAD precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the total of votes cast for president, using U.S. House Clerk official election returns data. Rug plots show the locations of state absolute margins.

Figure 17: Vote Counts for President, 1988



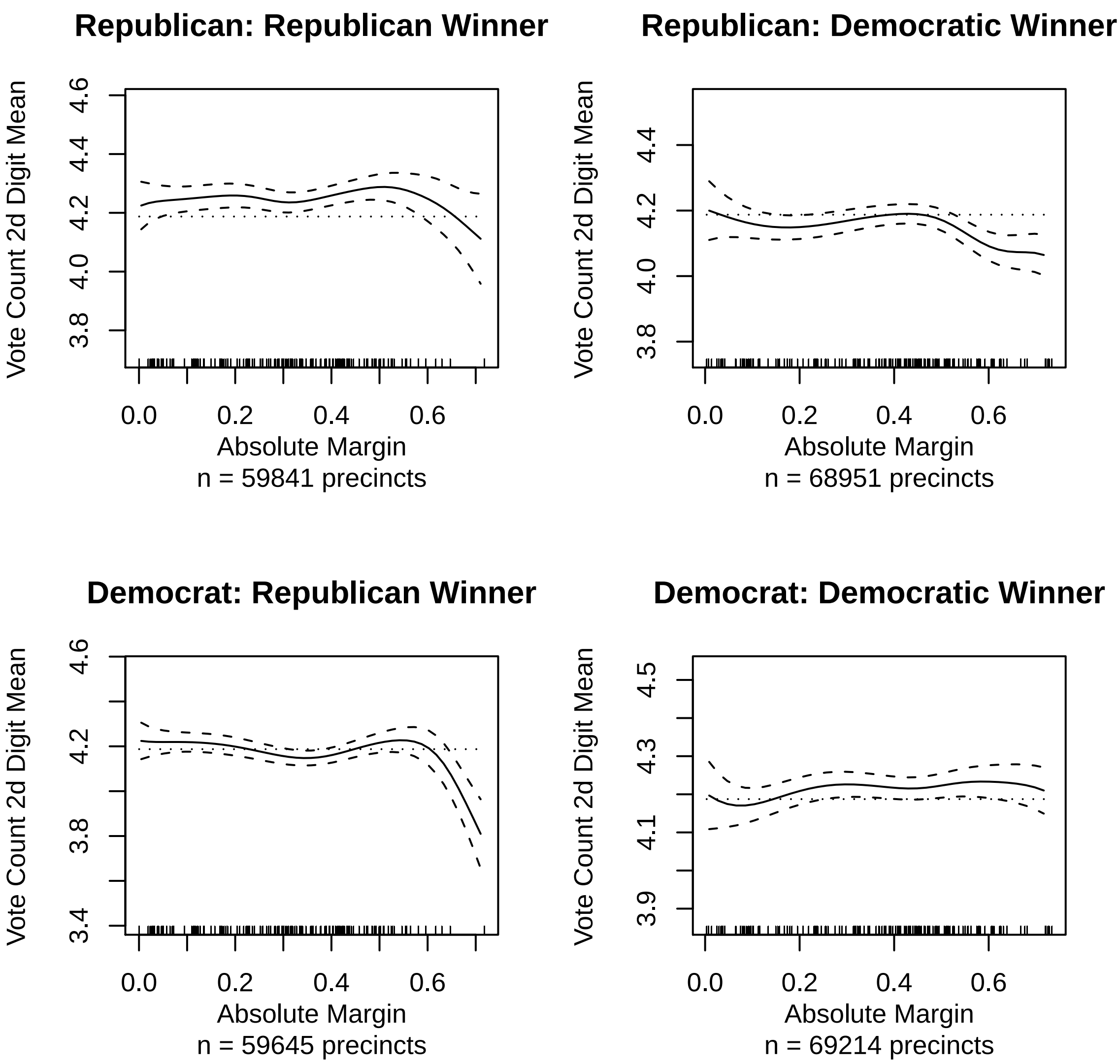
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on ROAD precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the total of votes cast for president, using U.S. House Clerk official election returns data. Rug plots show the locations of state absolute margins.

Figure 18: Vote Counts for United States Representative, 1988



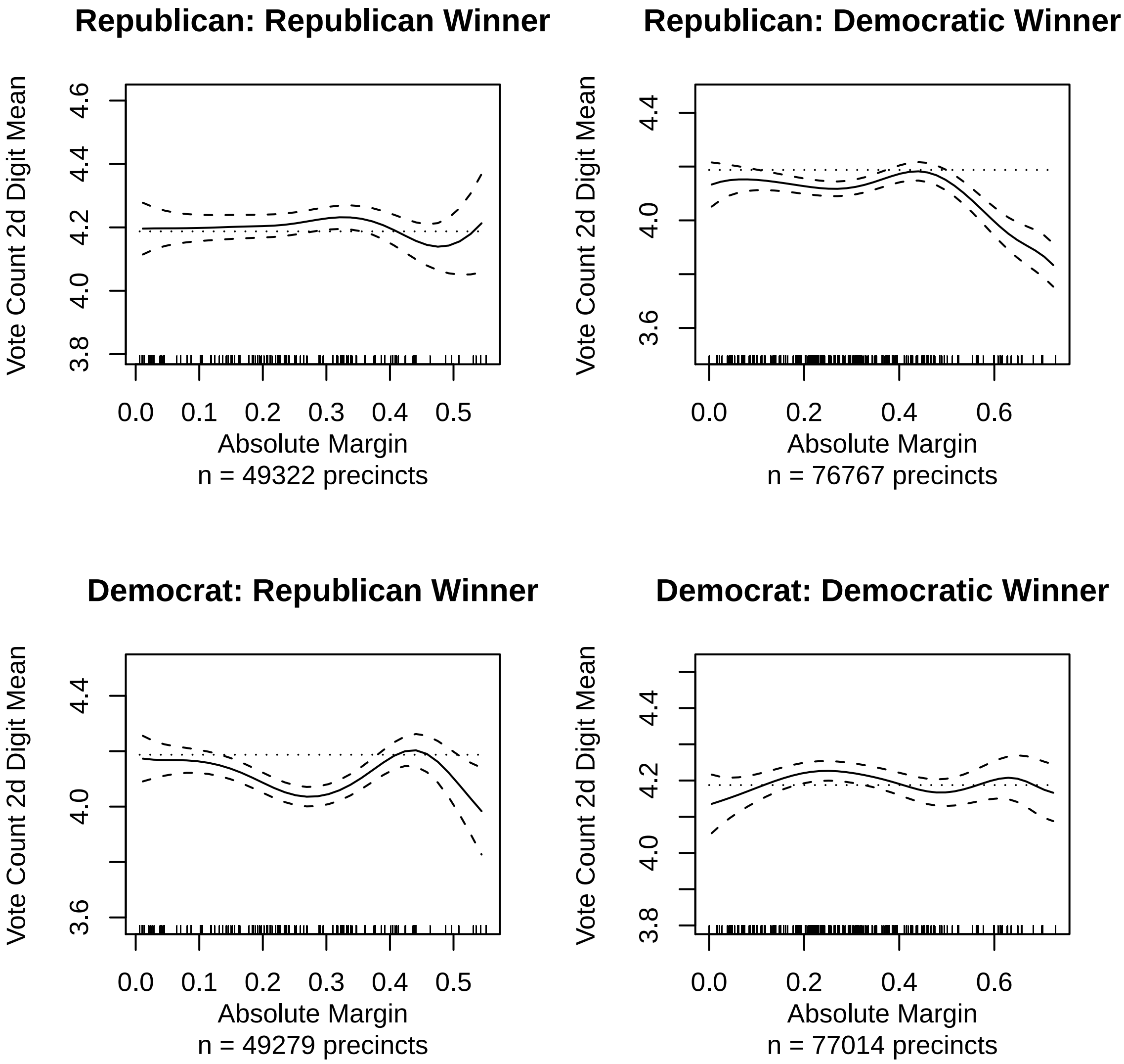
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on ROAD precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district absolute margins.

Figure 19: Vote Counts for United States Representative, 1986



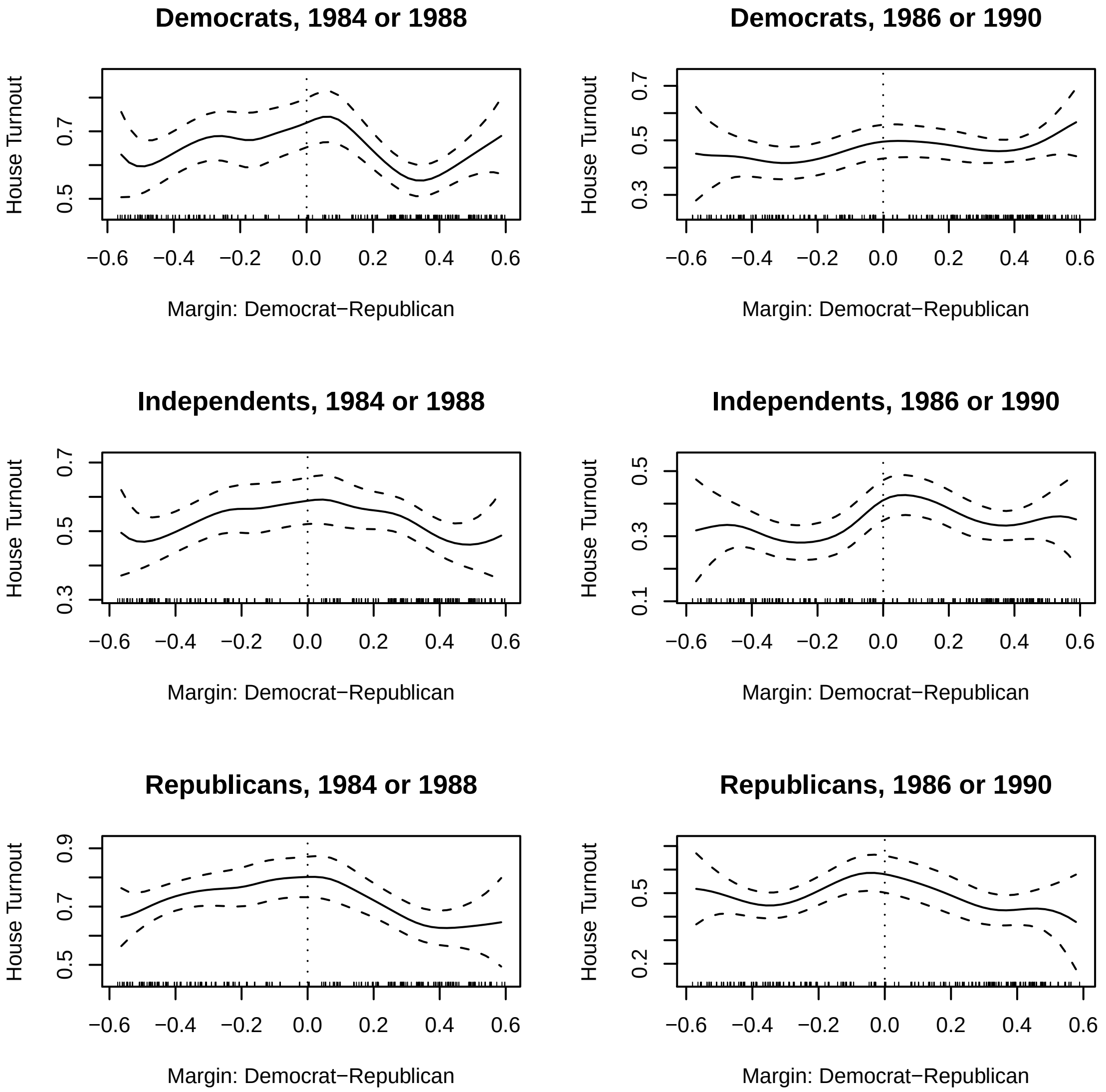
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on ROAD precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district absolute margins.

Figure 20: Vote Counts for United States Representative, 1990



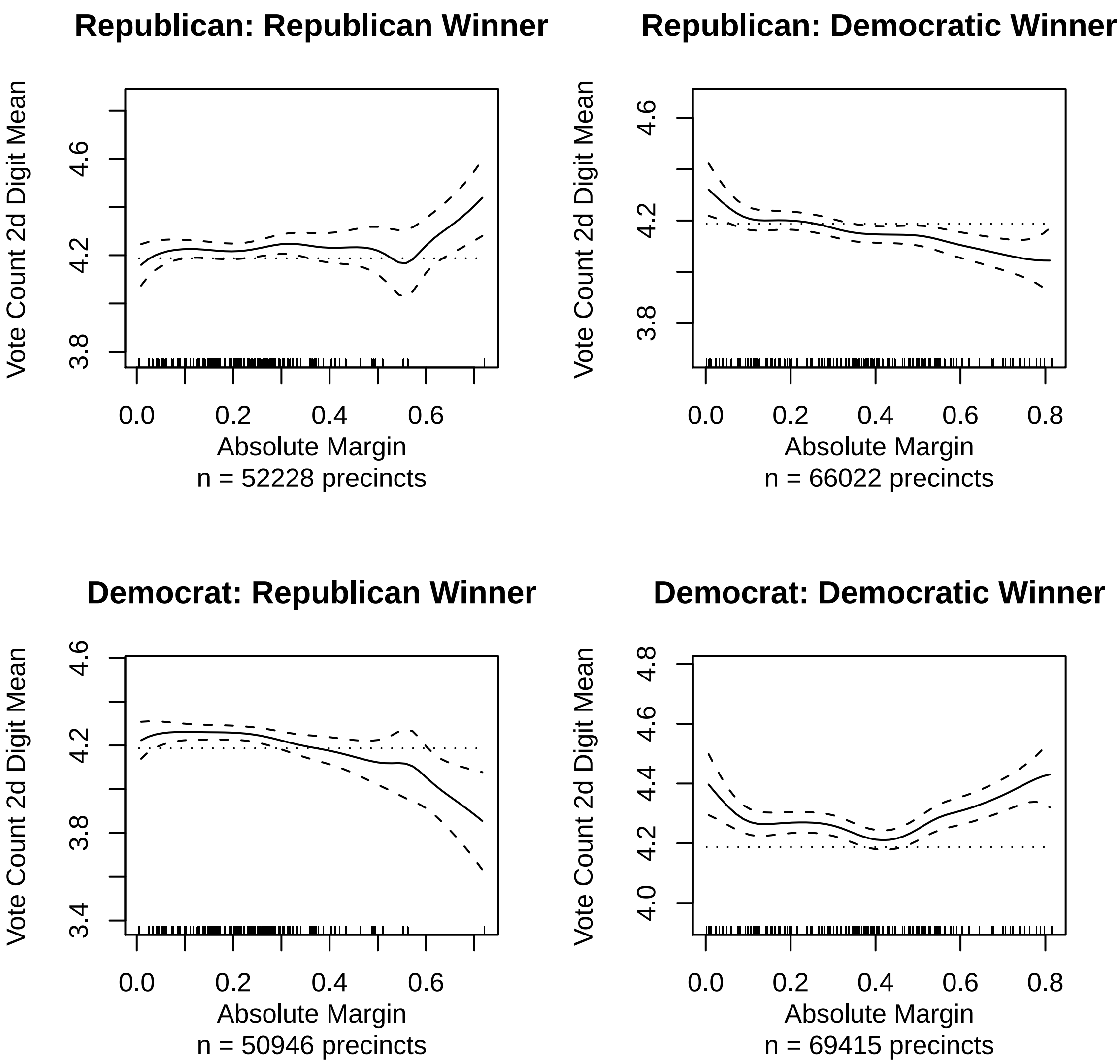
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on ROAD precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district absolute margins.

Figure 21: House Turnout by Margin by Party Identification, 1984–90



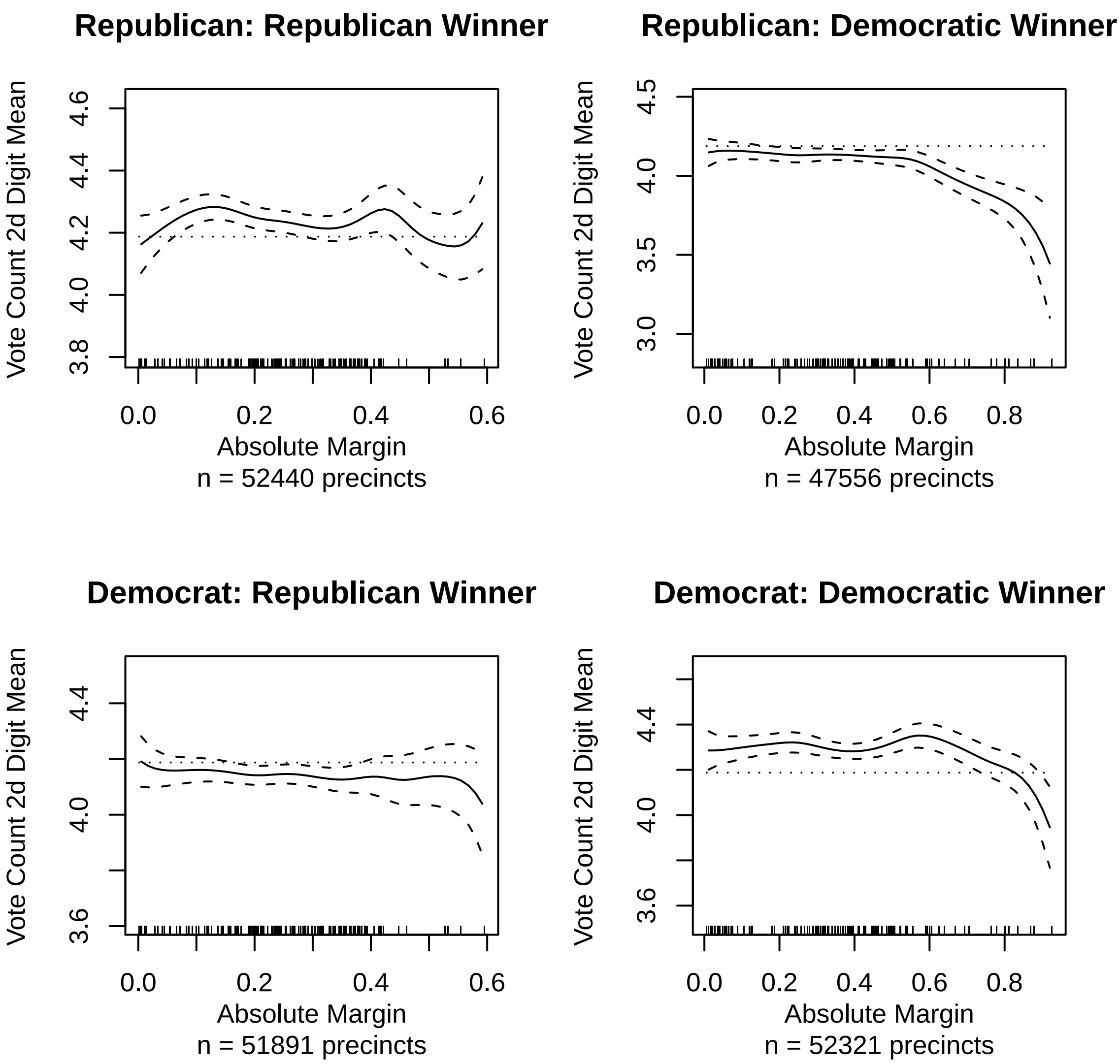
Note: “House turnout” is based on American National Election Studies data using“yes” responses to the question “Did you vote for a candidate for the U.S. House of Representatives?” with those who were validated as not having voted being counted “no.” “Margin” is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district margins.

Figure 22: Vote Counts for United States Representative, 2008



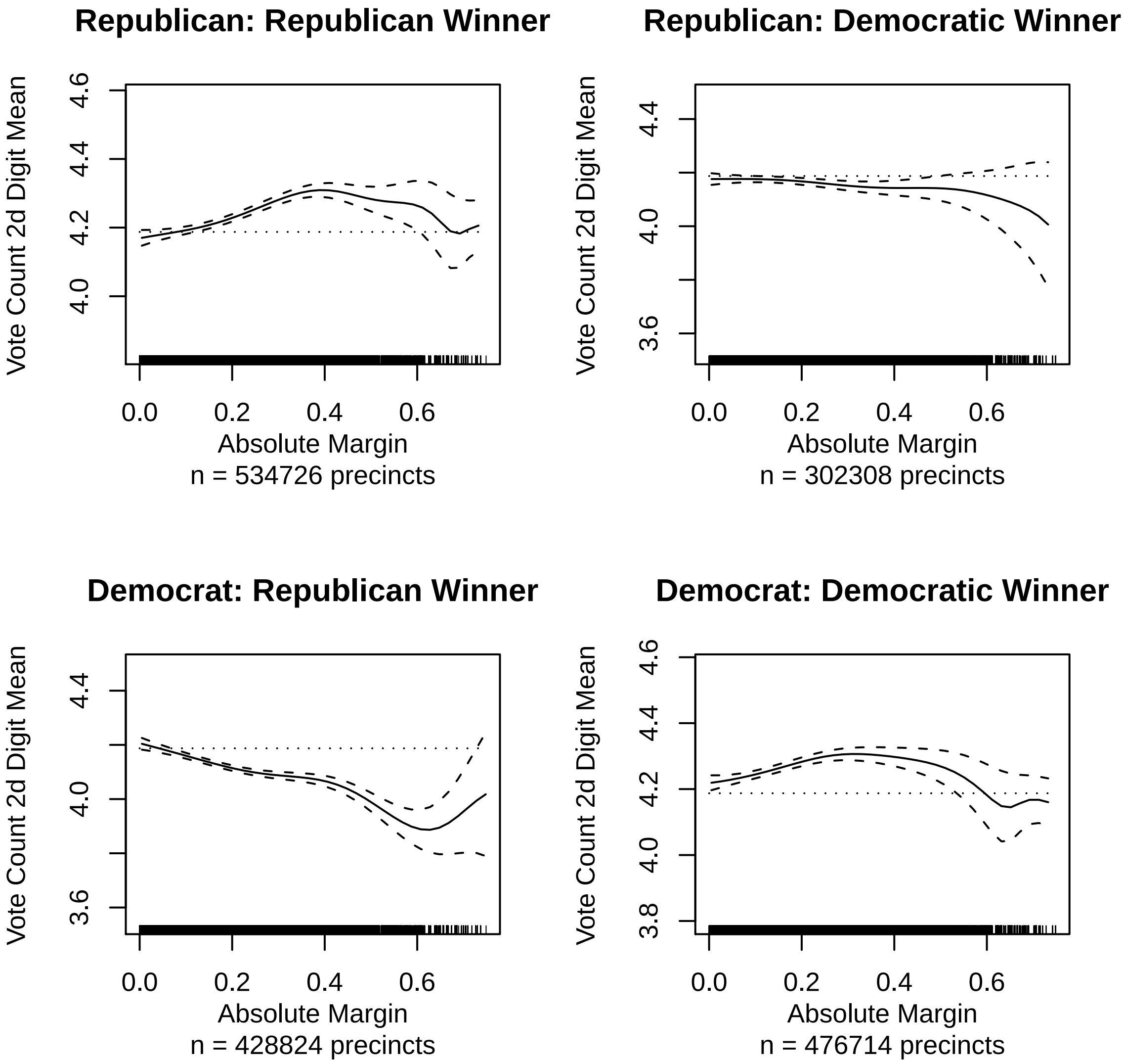
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district absolute margins.

Figure 23: Vote Counts for United States Representative, 2006



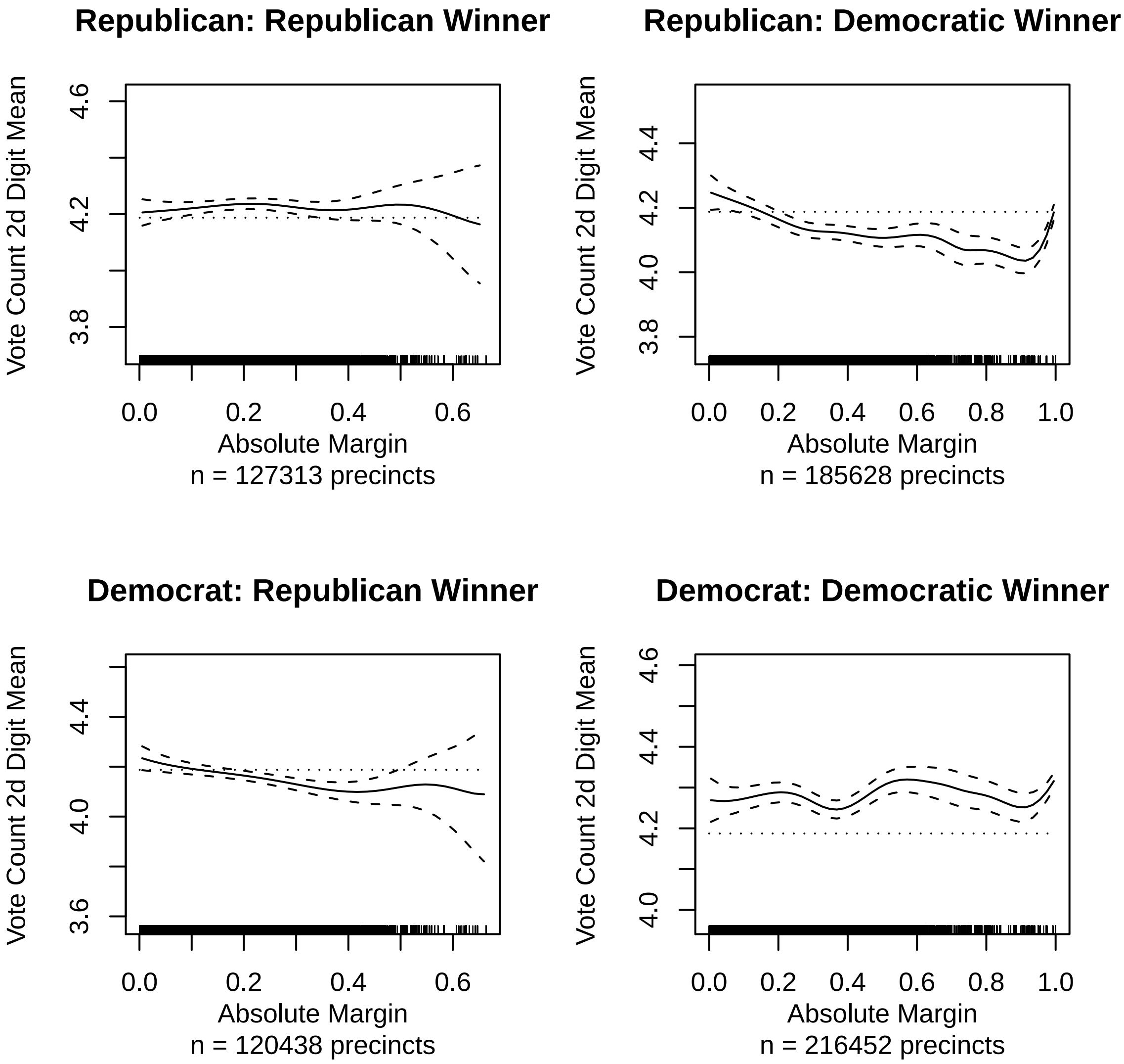
Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes, using U.S. House Clerk official election returns data. Rug plots show the locations of district absolute margins.

Figure 24: Vote Counts for State House and Senate, 1984–90



Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on ROAD precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes based on district totals computed from the precinct data. Rug plots show the locations of district absolute margins.

Figure 25: Vote Counts for State House and Senate, 2006–08



Note: Nonparametric regression curve (solid) with $\pm 1.96 \times$ s.e. curves (dashed). The dotted line shows the location of the second-digit mean expected under Benford's Law. "Vote Count 2d Digit Mean" is based on precinct data. "Absolute Margin" is based on the ratio of votes for the Democrat minus votes for the Republican divided by the sum of those two categories of votes based on district totals computed from the precinct data. Rug plots show the locations of district absolute margins.